Virtual Differential GPS & Road Reduction Filtering by Map Matching

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BIOGRAPHIES

Dr George Taylor, is a director of Quest Ltd, a software engineering company specializing in GPS and GIS R&D, and is also a lecturer in Geographical Information Science in The Department of Geomatics, University of Newcastle. Presently he is on a two year Royal Society Industry Fellowship to research GPS and digital mapping integration for vehicle tracking and navigation.

Professor Geoffery Blewitt recently moved to the University of Nevada from the University of Newcastle, where he was Professor of Space Geodesy. He previously worked at the NASA Jet Propulsion Laboratory where he led the research team which developed the GIPSY software which performs high precision GPS orbit determination and positioning. His current interests focus on the measurement and interpretation of earth dynamics.

ABSTRACT

A novel method of map matching using the Global Positioning System (GPS) has been developed for civilian use, which uses digital mapping data to infer the <100 meters systematic position errors which result largely from "selective availability" (S/A) imposed by the U.S. military. A method of rapidly detecting inappropriate road centrelines from the set of all possible road centre-lines that a vehicle may be travelling on has been developed. This is called the Road Reduction Filter. The S/A error vector is estimated in a formal least squares procedure as the vehicle is moving. This estimate can be thought of as a position correction from a "virtual" differential GPS (DGPS) base station, thus providing an autonomous alternative to DGPS for in-car navigation and fleet management. We derive a formula for "Mapped Dilution of Precision" (MDOP), defined as the theoretical ratio of position precision using virtual DGPS corrections to that using perfect DGPS corrections. This is shown to be purely a function of route geometry, and is computed for examples of basic road shapes. MDOP is favorable unless the route has less than a few degrees curvature for several kilometers. MDOP can thus provides an objective estimate of positioning precision to a vehicle driver. Precision estimates using MDOP are shown to agree well with "true" positioning errors determined using high precision (cm) GPS carrier phase techniques.

INTRODUCTION

The accurate location of a vehicle on a highway network model is fundamental to any in-car-navigation system, personal navigation assistant, fleet management system, National Mayday System (Carstensen, 1998) and many other applications that provide a current vehicle location, a digital map and perhaps directions or route guidance. Great many of these systems use the Global Positioning System (GPS) to initially determine the position of a vehicle.

The Global Positioning System has become the most extensively used positioning and navigation tool in the world. GPS provides civilian users with an instant (real-time) absolute horizontal positional accuracy of approximately 100 meters. Most of this error is due to intentional dithering of the GPS timing signal by the US Department of Defense, an effect known as Selective Availability (S/A). This level of positional accuracy is insufficient to ensure that a vehicle's location will correspond with the digitally mapped road on which the vehicle is travelling.

A number of methods have been successfully developed to significantly improve GPS accuracy, the most notable being differential GPS (DGPS). Real-time DGPS can improve positional accuracy down to 1 to 5m. However, the use of real-time DGPS in a moving vehicle requires additional data in the form of pseudorange corrections (computed errors in the satellite range measurements). Continuous reception of terrestrial radio transmissions or communications satellite broadcast is required to receive these corrections.

Often data can be combined from multiple sources integrating GPS with other navigational tools, such as attitude sensors such as the gyrocompass, vehicle odometer, flux gate compass and other dead reckoning methods. This use of multiple data sources again helps to correct for the error (noise) on the GPS position output. Multiple sensor data integration algorithms for vehicles are discussed by Mattos (1993). Dead reckoning produces the observed track by adding together the position vectors received from the sensor processor (Collier, 1990).

The fact that vehicles are generally constrained to a finite network of roads provides computer algorithms with digital information that can be used to correlate the computed vehicle location with the road network. This is known as map-matching. Many methods have been devised for map-matching (Scott, 1994) (Mallet et al., 1995). Our research has developed and tested an algorithm that utilizes GPS for the initial vehicle position and geometric information, computed from the digital road network itself, as the only other source of data for map-matching.

MAP-MATCHING METHODOLOGIES

Map-matching techniques vary from those using simple point data, integrated with optical gyro and velocity sensors (Kim, 1996), to those using more complex mathematical techniques such as Kalman Filters (Tanaka et al., 1990).

A semideterministic map-matching algorithm, described by French (1997), assumes that the vehicle is always on a predefined route or road network. The algorithm determines where the vehicle is along a route or within the network by determining instantaneous direction of travel and cumulative distance. This is a dead reckoning system, driven by interrupts from differential odometer sensors installed on the left and right wheels. The system uses the digital road map to check for correct left or right turns and to remove distance measurement. The positional error is converted into along-track and cross-track errors, allocating the first to the distance sensor and the second to the heading sensor errors (Mattos, 1993). For example, if the sensors indicate a 90 degree left turn and the digital mapping confirms this with the vehicle's current position, the distance count may be reset to zero. Dead reckoning and map-matching systems like this are often linked with GPS receivers through software filtering schemes such as Kalman filtering (Levy, 1997).

A mathematical framework for map-matching of vehicle positions using GPS is given by Scott (1994). The theoretical performance of a map-aided estimation process is assessed using error statistics to translate the raw positions onto the road network. However, Scott acknowledges that a key component of the map-aided estimator is correct road identification. All performance measures derived for the estimator are not applicable if the vehicle position has been projected onto the wrong road. This is true for performance measures of any map-matching algorithm.

Systems that use only geometric information must utilize the "shape" of line segments (road center-lines) that define the road network (Bernstein et al., 1998). A logical first step is to determine which road center-lines are candidates for the vehicle's true location. All road center-lines that cross the region of possible true position must be located, for example there are eight potential center-lines, highlighted in red, within the 80m region displayed in figure 1. This region will vary from a 100m radius circle around the computed raw/uncorrected GPS point position to a small error ellipse centered on the corrected position, with perhaps a semi-major and semi-minor axis

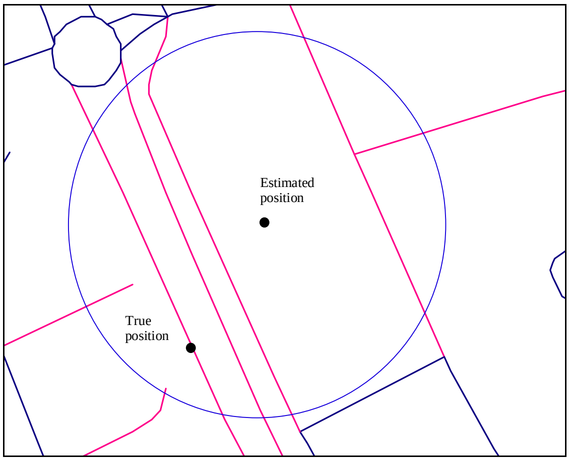

Figure 1. Potential road centre-lines

of 5m and 3m respectively. The shortest Euclidean distance from the GPS position to each of these road segments is computed and ordered by distance. The method used here must first calculate *A*, *B* and *C* for the implicit and normalized equation of a line through two points that define the road center-line (line segment):

$$Ax + By + C = 0 \tag{1}$$

Let the line be described as going from point *k* to *l*. If coordinates l_E, l_N are grid Easting and Northing for point *l*, and k_E, k_N are Easting and Northing for point *k*, then:

$$\begin{aligned} A &= k_N - l_N \\ B &= l_E - k_E \\ C &= k_E\, l_N - l_E\, k_N \end{aligned} \tag{2}$$

If *A* and *B* are both zero then it is a bad line definition, otherwise

$$\begin{aligned} A &= A\big/\sqrt{A^2 + B^2} \\ B &= B\big/\sqrt{A^2 + B^2} \\ C &= C\big/\sqrt{A^2 + B^2} \end{aligned} \tag{3}$$

Then calculate the shortest distance D from the GPS position p to line $Ax + By + C = 0$

$$D = \left| Ap_E + Bp_N + C \right| \qquad (4)$$

If D is equal to zero, then p is on the line. Also, if D is positive p is to the right of the line, or if D is negative p is to the left of the line joining k to l. This information may be of use to map-matching algorithms, and will be used later in this section.

It is not simply a matter of finding the line segment nearest to position p. This will often give an incorrect result. For example, in figure 1 the vehicle is on the highlighted road segment in the SW corner of the image, with the words road center-line next to it. Given a single GPS point position for the vehicle, it is not possible to determine which is the correct road segment, if there is more than one road segment in the neighborhood of the possible true position.

A better way to proceed is to match arcs defined by a series of GPS point positions $\{p_1, p_2, p_3 \ldots.., p_n\}$ with an arc defined by a set of points that define a partial road center-line $\{c_1, c_2, c_3 \ldots.., c_n\}$. One method used for matching two curves (arcs) is to use the distance between them.

If P and C are two such arcs the distance between them may be defined by the shortest distance between any pair of points taken from each arc, i.e.,

$$\|\mathrm{P} - \mathrm{C}\|_{\min} = \min_{p \in P, c \in C} \|p - c\| \qquad (5)$$

Bernstein et al. (1998) describes a curve (arc) matching algorithm, which uses a measure of the average distance between two arcs. This algorithm is implemented for map-matching in Princeton University's PULSAR Project. Arcs must be parameterized by using a function such as $p : [0,1] \rightarrow \mathrm{P}$, then

$$\|P - C\| = \int_0^1 \|p(t) - c(t)\| dt \qquad (6)$$

The problem with this algorithm is that it will only reliably detect the best match for arcs of the same length, which limits its ability to identify the correct arc in certain circumstances, e.g. slow moving vehicles.

Moments and moment invariants may potentially be used to match arcs. These properties of shapes are used in digital pattern recognition since they are independent of general linear transformations. Singer (1993) describes a method of moment expansion for linear objects (arcs) which may be used for arc comparisons. The moments associated with a line segment li can be written as

$$\mu(r,s) = \int_{li} x^r y^s dl \qquad (7)$$

Appropriate moments of two matching arcs would differ only by some small pre-determined tolerance.

Other methods are also used to reduce the number of potential road segments for the correct vehicle position. The topology of the road network may also be used. If the length of the connected route through a network from the present position on one particular road segment to the next position on another potential road segment is outside the possible range of distance traveled so far, that potential road segment is rejected. Carstensen (1998) has looked at the effects of filtering autonomous GPS points by number of satellites tracked, Dilution of Position (DOP) values or satellite geometry, and velocity and acceleration of the vehicle between positions. Filtering of potential road segments may be achieved using some of these measures and other criteria such as distance traveled and change of heading between vehicle positions.

RESEARCH METHODOLOGY

The method of map matching developed in this project is dependent on two main innovative techniques.

1. The computation algorithm of a least-squares estimation of the position provides complete control over which satellites will be used in the solution. This avoids step functions in the GPS positions as a result of the loss and gain of satellites. It enables the use of height aiding in the solution, i.e. one less unknown to solve, thus one less satellite required for the computation. Furthermore, it also enables the calculation and use of pseudorange corrections derived from the digital road network data (virtual DGPS).
2. A method of modeling Selective Availability (S/A) is introduced. Although in the long term S/A introduced error will reduce to approximately a Gaussian distribution (random error), in the short term (30sec) the effect of S/A can be viewed as a slowly varying bias (Scott, 1994). S/A will move the point position of a stationary receiver by approximately 10m to 30m per minute.

ROAD REDUCTION FILTER (RRF) MAP MATCHING ALGORITHM

1. At the very first epoch a **Raw** vehicle position is computed using all satellites available plus height aiding (height obtained from a Digital Terrain Model (DTM) and used to provide an extra equation in the least squares approximation computation, i.e. computation with a minimum of three satellites. For this first epoch all roads (road center-line segments), which are within

100m of the computed **Raw** position are selected. The point on each of the *n* road segments that computes the shortest distance to the **Raw** position, using equations (1) to (4), is selected as the first approximation of the true location of the vehicle, its **Ref** position. We can guarantee with 99% confidence that the vehicle is on one of these road segments. That is, we have *n* **Ref** positions that we can use to generate virtual DGPS corrections for use with the next epoch computed **Raw** position.

2. Virtual DGPS corrections for each satellite pseudorange are computed at each of the *n* **Ref** positions on each road segment for the current epoch. We have *n* different sets of virtual DGPS corrections.
3. The next epoch **Raw** position is computed.
4. Each of the virtual DGPS corrections (step 2) are added to the **Raw** position (step 3) to give *n* **Cor** positions on the *n* road segments.
5. Each of these *n* **Cor** positions is now snapped back onto the appropriate road-center lines to give *n* **Ref** positions. Go to step 2.

Instead of step 1, or at any time during the observations, the position of the vehicle may be provided by user intervention. This would result in only one road center-line for computation. That is until a road junction is encountered, when again as in step 1, a number of road center-lines would have to be considered.

This process from steps 2 to 5 is repeated for each new epoch. At each epoch for each of the n road segments the following data is computed and stored:

Raw distance	-	previous **Raw** to current **Raw** position.
Cor distance	-	previous **Cor** to current **Cor** position.
Ref distance	-	previous **Ref** to current **Ref** position.
Raw bearing	-	previous **Raw** to current **Raw** position.
Cor bearing	-	previous **Cor** to current **Cor** position.
Ref bearing	-	previous **Ref** to current **Ref** position.
S/A error	-	Estimation of Selective Availability error, (described later)

This data is held for the last 30 epochs for each road center line processed.

DETERMINING THE CORRECT ROAD CENTER-LINE

The task of the RRF is to determine the correct road center-line segment from the set of those possible, or conversely, which road center-lines to reject.

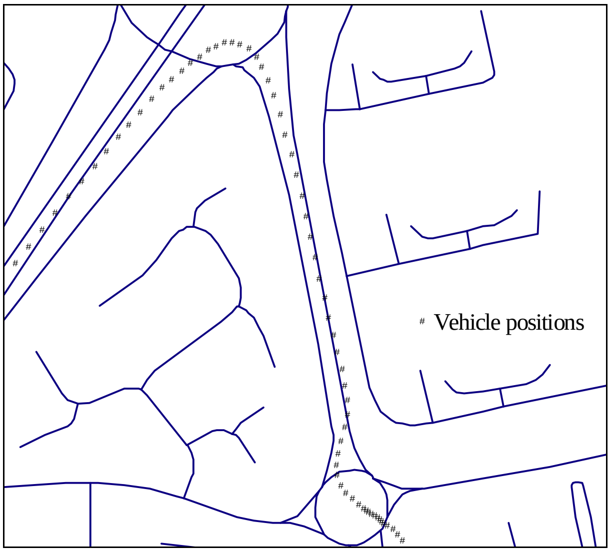

Figure 2. Raw GPS positions and road centre lines

The trajectory defined by **Raw** GPS positions computed from observations taken by a receiver in a moving vehicle is correlated with the shape of the digitized road center line on which the vehicle is travelling. This correlation is high if the vehicle is traveling at high speed, and low if the vehicle is traveling at low speed, because at high speed, the S/A bias changes less per unit distance traveled.

By calculating values for distance traveled, bearing, velocity and acceleration between epochs for **Raw** positions and comparing these values with equivalent Ref positions, it is possible to filter out many road potential center-lines. There is a very high correlation if we compare bearing and distance between successive **Raw** positions and the bearing and distance between successive **Ref** positions on the correct road center-line, if the vehicle is moving. We can calculate the following errors for each epoch:

Distance error = difference between **Raw** distance and **Ref** distance
Bearing error = difference between **Raw** bearing and **Ref** bearing

Figures 3 and 4 compare errors for eight different series of **Ref** positions, i.e., eight different road center-lines, over a period of 30 epochs (1 second interval). If a predefined tolerance value is set, perhaps 5m, it can be seen in figure 3 that series 2, 6, 7 and 8 can quickly be filtered out at almost any epoch. Series 3 and 4 are not as easy to identify for filtering out. Series 1 is even more difficult to remove. Series 5 is the set of Ref positions on the correct road center, this has a maximum error of 5m at epoch 10, where the vehicle is driven straight on at a roundabout, see figure 1. Series 3, 4, 1 and 5 are in fact positions from the four parallel road center-lines displayed in red in figure 1. The **Raw**, **Cor** and **Ref** positions for this correct road center-line are shown in figure 5.

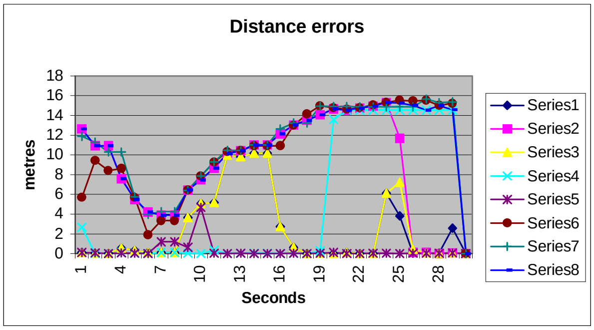

Figure 3. Eight road center-lines tracked

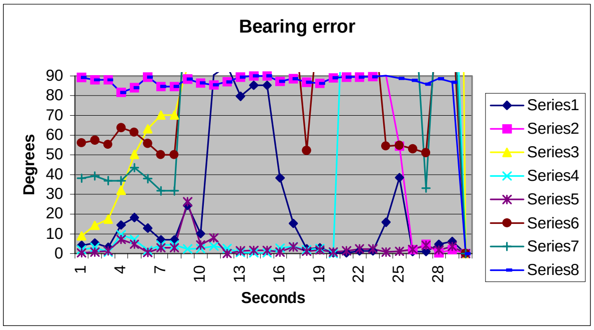

Figure 4. Eight road center-lines tracked

Figure 5. Correct road center-line only

LEAST SQUARES ESTIMATION OF SELECTIVE AVAILABILITY (S/A)

One problem with the approach presented above is that Selective Availability (S/A) has errors of up to 100m, so while it may be possible at a particular point in time to correctly identify the correct road, the position along the road may be in error by up to 100m. This "along track error" cannot be resolved for a straight road, but it can be resolved if the road changes direction, or if the vehicle turns a corner. We now present a more formal method of computing the virtual DGPS correction, which is then integrated with the Road Reduction Filter (RRF). The advantage of formal methods is that quality measures can be derived and used to place confidence bounds for rigorous decision making (for example, to reject road center-lines that fail a particular hypothesis test). Formal methods also provide insight into the relative importance of factors, which can improve the procedure (e.g., data rates and road geometry).

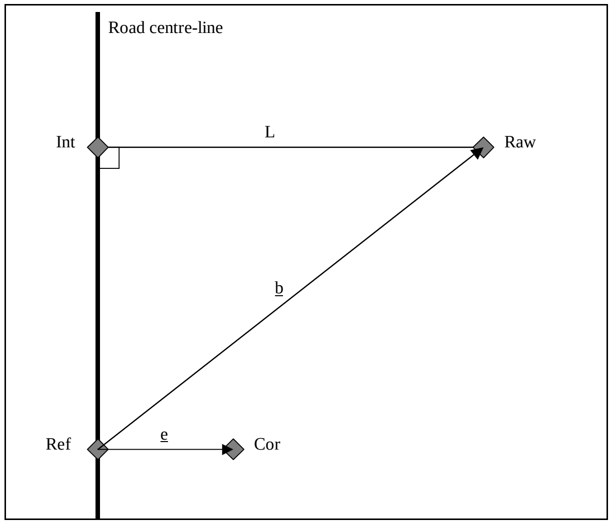

Figure 6. Error vector for Selective Availability (S/A)

Figure 6 displays a GPS position at a single epoch. We can consider the vector b to be the error vector (S/A vector) from the true vehicle position on the road center-line at grid position **Tru**(E_{Tru}, N_{Tru}) to the uncorrected position computed from GPS at **Raw**(E_{Raw}, N_{Raw}). The perpendicular distance from the **Raw** position to the road center-line at **Int**(E_{Int}, N_{Int}) is given as L. The road centre line for this purpose is defined by extending the line segment, which joins previous **Ref** to current **Ref**. The first approximation of the **Tru** position is the **Ref** position, which (as explained in step 5 above) was obtained by snapping the **Cor** position (the **Raw** position corrected using virtual DGPS corrections) onto the closest point on the road center-line. Furthermore, the observed perpendicular distance from **Raw** position to the road center line at **Int** is given by L where:

$$L = \pm\sqrt{(E_{\text{Raw}} - E_{\text{Int}})^2 + (N_{\text{Raw}} - N_{\text{Int}})^2} \quad (8)$$

The positive root of L is taken if the raw point lies to the right of the center-line, and the negative root if it lies to the left. As L has a sign, it may be better described as a "cross track coordinate" rather than a distance.

Here, L is introduced as a "measurement" which can be modeled geometrically. The model that best fits a series of these measurements provides an estimate of the S/A vector, $\underline{b}$. Consider the unit vector $\underline{\hat{e}}$ which points normal to the road center line (and to the right of the road) at the **Ref** position: The cross track coordinate L in equation (8) may also be modeled (computed) the dot product of the two vectors $\underline{b}$ and $\underline{\hat{e}}$

$$\underline{b} = b_E\underline{E} + b_N\underline{N} \qquad (9)$$
$$\underline{\hat{e}} = r_N\underline{E} - r_E\underline{N}$$

where $\underline{E}$ and $\underline{N}$ are unit vectors pointing in the East and North directions, and r_E and r_N are the direction cosines of a road segment at the **Ref** position are computed using the RRF algorithm for A and B in equations 2 and 3. For analytical purposes later (equation (23) (24) and (25)), it is convenient to write them in term of ϕ, the bearing (clockwise azimuth from North) of the road segment.

$$r_E = \sin\phi \qquad (10)$$
$$r_N = \cos\phi$$

Therefore we have the following observation equation, where the left side is measured, and the right side is modeled, and includes an unknown term v which absorbs random position errors (excluding S/A):

$$\begin{aligned} L &= \underline{b}.\underline{\hat{e}} + v \qquad (11)\\ &= b_E r_N - b_N r_E + v \end{aligned}$$

We get such an equation each time we have a GPS raw estimated of position. If we consider n successive GPS raw estimates over a time period where the S/A vector $\underline{b}$ can be assumed to be approximately constant, we have:

$$\begin{aligned} L_1 &= b_E r_{N1} - b_N r_{E1} + v_1 \\ L_2 &= b_E r_{N2} - b_N r_{E2} + v_2 \\ L_3 &= b_E r_{N3} - b_N r_{E3} + v_3 \qquad (12)\\ &\vdots \\ L_n &= b_E r_{Nn} - b_N r_{En} + v_n \end{aligned}$$

In practice, b varies at a level comparable to a road width over 30 seconds, hence for GPS raw estimates every second, n can have a value of 30. This can be written in matrix form:

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \\ . \\ . \\ L_n \end{pmatrix} = \begin{pmatrix} r_{N1} & -r_{E1} \\ r_{N2} & -r_{E2} \\ r_{N3} & -r_{E3} \\ \vdots & \vdots \\ r_{Nn} & -r_{En} \end{pmatrix} \begin{pmatrix} b_E \\ b_N \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ . \\ . \\ v_n \end{pmatrix} \qquad (13)$$

which can be written compactly as,

$$\mathbf{L} = \mathbf{Ax} + \mathbf{v} \qquad (14)$$

We now apply the principles of least squares analysis (Blewitt, 1997), which minimizes the sum of squares of estimated residuals, giving the following solution for (b_E, b_N):

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{L} \qquad (15)$$

The estimated residuals (misfit of model to the data) are given by:

$$\hat{\mathbf{v}} = \mathbf{L} - \mathbf{A}\hat{\mathbf{x}} \qquad (16)$$

which can then be checked to assess model fidelity.

Least squares assume that the errors v_i are random with zero mean expected value (i.e., some will be positive, some negative). This is a reasonable model for GPS pseudorange measurement error, but is not a good model for persistent systematic effects such as atmospheric delay and errors in satellite positions computed from the Navigation Message. However, such systematic effects will be absorbed by the S/A vector estimate. Note that such persistent effects are not only common mode to a single receiver's measurements over a short time period, but would also be in common to all GPS stations in the local area. Clearly, the estimated S/A vector $\hat{\mathbf{x}}$ is equivalent to a "position correction" which could be provided by a local DGPS base station. We therefore call our technique "Virtual DGPS", because it does not require data from another GPS base station, and yet it provides the same type of information. For purposes of this paper, we will retain the term "S/A vector" to emphasize that S/A is the dominant effect, and it does motivate such technique development.

Note that the GPS data and the digital map data have been incorporated into this formal scheme through the "measurement" of L. An advantage of taking such a formal approach to map matching can therefore be seen as the quantification of expected errors, which can in turn be used to narrow down the search for possible positions. For example, alternative hypotheses where a vehicle may have taken one of three roads at a junction can be assessed in terms of the level of estimated residuals, as compared to the level of expected errors.

The modeled error in the determination of the S/A vector can be found from the covariance matrix, which can then be used to plot a confidence ellipse within which the true value of S/A bias can be expected to lie. The covariance matrix is computed as:

$$C = \sigma^2 \left(\mathbf{A}^{\mathbf{T}}\mathbf{A}\right)^{-1} \tag{17}$$

The constant σ^2 represents the variance in raw GPS positions, excluding the effects of S/A and common mode errors. In other words, σ should equal the standard deviation in raw GPS positions if perfect DGPS corrections were used to remove the effects of S/A and other non-random common mode errors. Its value tends to be dominated by multipathing around the vehicle, and varies with the geometry of the satellite positions, an effect known as "horizontal dilution of precision" (HDOP). Typical values are at the meter level. One possibility is to use the estimated residuals themselves to estimate the level of σ. However, this would be inadvisable because we intended to use C to test the significance of high levels of residuals, which would have created a circular argument.

QUANTIFYING ROAD GEOMETRY: "MAPPING DILUTION OF PRECISION (MDOP)"

The equation (17) given above for the computation of the covariance matrix leads to an elegant method of quantifying road geometry as to its suitability for estimating selective availability on-the-fly.

First note that the least squares method assumes that the "cofactor matrix" $\left(\mathbf{A}^{\mathbf{T}}\mathbf{A}\right)^{-1}$ exists. It is necessary but not sufficient requirement that $n \geq 2$. If the two **Ref** positions are collinear (the road is perfectly straight) then a third position is required that is not collinear. In the work here n = 30. We now explore how the cofactor matrix can be interpreted, and how it is related to the shape of the road.

The diagonal elements of the cofactor matrix can each be interpreted from equation (17) as the ratio of the error squared in estimated S/A vector component to the expected error squared of a single GPS position in the case that an ideal DGPS position correction were used. To obtain a single number that relates to standard deviation of position instead of variances and covariances, we follow the example of classic GPS theory by which the square root of the trace of the cofactor matrix is taken as a "Dilution of Precision" (DOP) value. We therefore define "Correction Dilution of Precision" (CDOP) as:

$$\mathrm{CDOP} = \sqrt{\mathbf{Tr}\left(\mathbf{A}^{\mathbf{T}}\mathbf{A}\right)^{-1}} \tag{18}$$

From the definition of matrix A in equation (13), we can write CDOP in terms of the direction cosines at each of the sampled points on the road. Starting with the cofactor matrix:

$$\left(\mathbf{A}^{\mathbf{T}}\mathbf{A}\right)^{-1} = \begin{pmatrix} \sum_n r_{Ni}^{\ 2} & -\sum_n r_{Ei} r_{Ni} \\ -\sum_n r_{Ei} r_{Ni} & \sum_n r_{Ei}^{\ 2} \end{pmatrix}^{-1} = \frac{\begin{pmatrix} \sum_n r_{Ei}^{\ 2} & \sum_n r_{Ei} r_{Ni} \\ \sum_n r_{Ei} r_{Ni} & \sum_n r_{Ni}^{\ 2} \end{pmatrix}}{\sum_n r_{Ei}^{\ 2} \sum_n r_{Ni}^{\ 2} - \left(\sum_n r_{Ei} r_{Ni}\right)^2} \tag{19}$$

Therefore equations (19) into (18) gives

$$\mathrm{CDOP} = \left(\frac{\sum_n r_{Ni}^{\ 2} + \sum_n r_{Ei}^{\ 2}}{\sum_n r_{Ei}^{\ 2} \sum_n r_{Ni}^{\ 2} - \left(\sum_n r_{Ei} r_{Ni}\right)^2}\right)^{1/2} \tag{20}$$

From equations (10), the numerator is simply n, so the whole formula can be reduced to:

$$\mathrm{CDOP} = n^{-1/2}\left(\overline{r_{Ei}^{\ 2}}\ \overline{r_{Ni}^{\ 2}} - \overline{r_{Ei} r_{Ni}}^{\,2}\right)^{-1/2} \tag{21}$$

where the overbars denote averaging over the section of road (for which S/A is assumed to be approximately constant). CDOP therefore depends on road geometry, and will be inversely proportional to the number of GPS measurements *n* taken over a fixed time interval. With enough measurements and with sufficient change in road direction, it is possible to reduce CDOP to <1, in which case S/A is no longer a limiting error source.

Note that GPS data recording should be sufficient to sample any detail in road shape that is present in the digital map. It is therefore preferable to record GPS data at a high rate, e.g., 1 per second. Going at higher rates than this will not help particularly, because of time correlated errors in multipathing, and because at this rate, the road is approximately straight between points. Where there is detailed road shape the rate of sampling will increase naturally due to necessary a reductions in vehicle velocity.

A related quality measure is “Mapping Dilution of Precision,” (MDOP) which we define as the ratio of position precision using virtual DGPS corrections to that using perfect DGPS corrections. In this case, we assume that if *n* is much greater than 1, then the virtual DGPS correction (i.e., the S/A vector) is uncorrelated with the error in any single data point. Therefore the corrected position will have a variance equal to the variance in the perfect case plus the variance in the correction. As this is to be divided by the variance in the perfect case, the result is:

$$\begin{aligned}\mathrm{MDOP} &= 1 + \mathrm{CDOP} \\ &= 1 + \sqrt{\mathbf{Tr}\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}}\end{aligned} \tag{22}$$

This measure is particularly useful because:

- it is easily interpreted as a “level of degradation” in precision as a result of not using a perfect DGPS base station.
- it can be tested for validity under controlled conditions.

As we shall describe, testing was carried out using an ultra precise GPS method (e.g., carrier phase positioning) to determine the true level of corrected position errors, and then compare this with the errors obtained by applying a near-perfect DGPS correction. The point is that equation (22) can be computed easily in real time (even ahead of time!) by simply knowing the road shape.

Note that MDOP is always greater than 1 because comparison is made with perfect DGPS. It is worth keeping in mind that no DGPS system is perfect; hence MDOP > 1 does not necessarily mean that real DGPS will give better results than virtual DGPS.

MDOP FOR BASIC ROAD SHAPES

From equations (10), (21) and (22), we can write MDOP analytically in terms of the direction cosines of the vector normal to the road.

$$\mathrm{MDOP} = 1 + n^{-1/2}\left(\overline{\sin^2\phi}\;\overline{\cos^2\phi} - \overline{\sin\phi\cos\phi}^{\,2}\right)^{-1/2} \tag{23}$$

This equation can be rearranged into the following form:

$$\mathrm{MDOP} = 1 + 2n^{-1/2}\left(1 - \overline{\cos 2\phi}^{\,2} - \overline{\sin 2\phi}^{\,2}\right)^{-1/2} \tag{24}$$

The first thing to note about MDOP is that it takes on the following maximum (worst case) and minimum (optimum) values:

$$\mathrm{MDOP}_{\max} = \infty \qquad ;\ \phi = \text{constant}$$

$$\mathrm{MDOP}_{\min} = 1 + 2n^{-1/2} \qquad ;\ \overline{\cos 2\phi} = \overline{\sin 2\phi} = 0$$

The maximum condition is satisfied for a straight road. As we shall see, the minimum condition is satisfied for the simple case of a right-angled bend. Keeping in mind the definition of MDOP, we see that S/A ceases to be a dominant error source when MDOP ≤ 2, which the above equation satisfies when using 4 GPS measurements around a right-angled bend. As more measurements are introduced, MDOP approaches 1, which implies that positioning is as good as using a perfect DGPS system.

Equation (24) can be easily computed for any road using a graphical interpretation of the term we call the “path closure ratio”:

$$S(\vartheta) = \overline{\cos\vartheta}^{\,2} + \overline{\sin\vartheta}^{\,2} \tag{26}$$

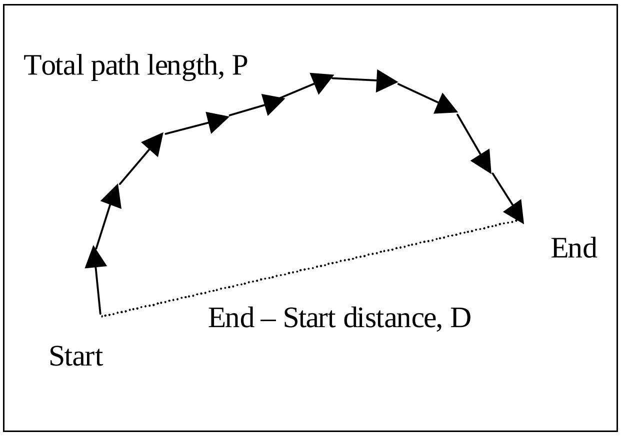

Figure 7. Path constructed of unit vectors.

Consider a path constructed using segments *i* each of equal length and with bearing ϑ_i (figure 7). The path closure ratio *S* can be shown to be equal to the square of the ratio of straight-line distance between the starting and end points *D* to the total path length *P*:

$$S(\vartheta_i) = \left(P/D\right)^2 \tag{27}$$

Obviously, S ranges from 0 to 1. We can therefore take our digital map of the road, and transform it to a path where all of the path segments have double the bearing of the real road, and where each road segment between GPS points are mapped into segments of equal length. We can then compute MDOP as follows:

$$\begin{aligned}\mathrm{MDOP} &= 1 + 2n^{-1/2}\left(1 - S(2\phi)\right)^{-1/2} \\ &= 1 + 2\Big/\sqrt{n\left(1 - \left(P/D\right)^2\right)}\end{aligned} \tag{28}$$

Note that a path of fixed length *P* is therefore equivalent to a road section covered in a fixed amount of time (because GPS data are recorded at equal intervals). So for a fixed amount of time, the path which ends closest to the starting point produce a smaller value of *S*, and a smaller (more favorable) value of MDOP.

This graphical method is so powerful, that results can be visualized without any computation (Figure 8). For example, sharp right-angled bend in a road will map onto a path which doubles back on itself, reducing *S* to zero, and hence producing the minimum value of MDOP. A road which gently sweeps though 90 degrees will map onto a path which heads back in the opposite direction, but is displaced by some distance, and therefore will produce good, but not optimum results. A road, which moves in a semi-circle (e.g., around a large roundabout), will map into a path, which is a complete circle, and hence will produce optimum results.

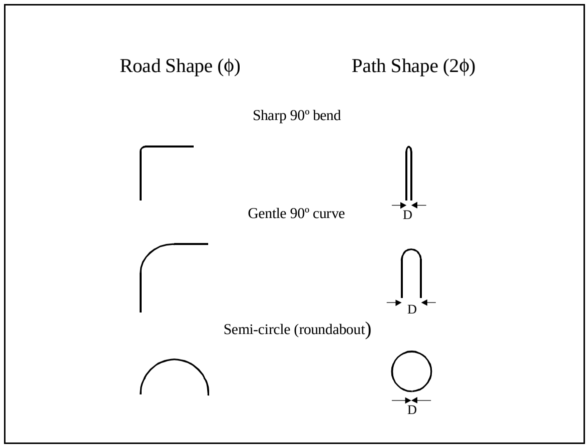

Figure 8. Basic road shapes can be transformed into path shapes with twice the curvature, which can then be interpreted in terms of favorable geometry (MDOP).

Table 1 summarizes the results for the computation of the path closure ratio $S(2\phi)$ for various road shapes which can then be inserted into equation (28) to find the appropriate MDOP value. Also given is the value of *n*, which would be required to bring the MDOP value < 2. We call this number the "resolution time" *T*, since it tells us how many data intervals are required to bring S/A to a level below that expected from random position errors. Under the assumption that we use 1 second GPS data, *T* is in seconds.

Road Shape Description	Path Closure Ratio $S(2\phi)$	Mapping Dilution of Precision, MDOP	Resolution Time T (sec)
Instant bend, angle α	$\cos^2\alpha$	$1+2/\sin\alpha\sqrt{n}$	$4/\sin^2\alpha$
Instant bend, 90º	0	$1+2/\sqrt{n}$	4
Instant bend, 45º	0.5	$1+2.8/\sqrt{n}$	8
Instant bend, 20º	0.88	$1+5.8/\sqrt{n}$	34
Instant bend, 10º	0.97	$1+11.5/\sqrt{n}$	133
Smoothest curve, α	$\sin^2\alpha/\alpha^2$	$1+2/\sqrt{(1-\sin^2\alpha/\alpha^2)n}$	$4/(1-\sin^2\alpha/\alpha^2)$
Smoothest curve, 90º	$4/\pi^2=0.41$	$1+2.6/\sqrt{n}$	7
Smoothest curve, 45º	$8/\pi^2=0.81$	$1+4.6/\sqrt{n}$	22
Smoothest curve, 20º	0.96	$1+10.0/\sqrt{n}$	100
Smoothest curve, 10º	0.99	$1+19.9/\sqrt{n}$	396

Table 1. Quality Measures Associated with Various Road Geometries for VDGPS

CONCLUSIONS

From Table 1 we can see that S/A can be resolved to within the expected random error of perfect DGPS for all except the slightest of change in road geometry. Problems begin to arise with roads which curve by only 20 degrees within the time frame that S/A is assumed to be constant (~30 sec for road navigation), although even 10 degrees are sufficient provided the bend is effectively instantaneous. We therefore conclude that only if roads are straighter than 10-20 degrees during a 30 second driving period (i.e., 0.4-1 km in typical driving conditions) will Virtual DGPS be significantly worse than DGPS. However, the full precision of DGPS is certainly not required for finding the correct road center-line, so these numbers are in any case extremely conservative for that purpose. In summary, we expect on firm theoretical grounds that RRF and Virtual DGPS techniques combined to be as good as DGPS for correct road center-line identification in almost any possible circumstance. This has the distinct advantage of being a completely self contained system, requiring no radio communication for differential corrections and continuous data provision. Furthermore, because the computation of the estimated GPS receiver position is part of the RRF and a digital terrain model derived height aiding is used in the solution, only three satellites are necessary for a solution.

Envisaged further work will include extensive field testing of the combined Virtual DGPS and RRF approach to vehicle tracking. Moreover, an investigation of other techniques to reduce the number of satellites required for a solution will be made. Bullock et al. (1996), examined two satellite tracking for urban canyons and map matching requires only a two dimensional position. Furthermore, an attempt to implement hierarchical spatial reasoning techniques (Car, 1997) will be made to improve the efficiency of the RRF algorithm.

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REFERENCES

Bernstein, D. and Kornhauser, A., (1998), Map matching for personal navigation assistants, 77th Annual meeting, The Transport Research Board, Jan 11-15, Washington, D.C.

Blewitt, G., (1997), Basics of the GPS Technique: Observation Equations, chapter in book Geodetic Applications of GPS, The Swedish Land Survey.

Bullock, J.B., Geier, G.J., King, T.M. and Kennedy, H.L., (1996), Effectiveness of Two Satellite Tracking in Urban Canyon Environments, Proceedings of ION GPS-96, Sixth International Technical Meeting of the Satellite division of The institute of Navigation, Kansas City, Missouri, pp 1699 – 1707.

Car, A., (1997). Hierarchical Spatial Reasoning: Theoretical Consideration and its Application to Modeling Wayfinding. A. U. Frank and P. Haunold, Geoinfo Series. Vienna, Austria. ISBN 3-901716-10-6

Carstensen, Jr, L.W., (1998), GPS and GIS: Enhanced Accuracy in Map Matching through Effective Filtering of Autonomous GPS Points, Cartography and Geographical Information Systems, Vol. 25, No. 1, pp51-62

Collier, C., (1990), In-vehicle route guidance systems using map matched dead reckoning, Proceedings IEEE Position and Navigation Symposium, pp 359 – 363.

French, R.L., (1997), Land Vehicle Navigation and tracking, Global Positioning System: Theory and Applications Vol. II, pp 275 – 301.

Kim, J.S., Lee, J.H., Kang, T.H., Lee, W.Y. and Kim, Y.G., (1996), Node based map matching algorithm for car navigation system, Proceedings of the 29th ISATA Symposium, Florence, Vol. 10, pp 121 – 126.

Levy, L.J., (1997), The Kalman Filter: navigation's Integration Workhorse, GPS World, September, pp 65 – 71.

Mallet, P. and Aubry P., (1995), A low-cost localisation system based on map matching technique, International Conference on Intelligent Autonomous Systems, Karlsruhe, Germany, pp 72 – 77.

Mattos, P.G., (1993), Intelligent Sensor Integration Algorithms for Vehicles, Proceedings of ION GPS-93, Sixth International Technical Meeting of the Satellite division of The institute of Navigation, Salt Lake City, Utah, pp 1591 – 1597.

Scott, C., (1994), Improving GPS Positioning for Motor-Vehicle through map matching, Proceedings of ION GPS-94, Salt Lake City, Utah, pp 1391 – 140.

Singer, M.H., (1993), A general approach to moment calculations for polygons and line segments, Pattern Recognition, Vol. 26, No. 7, pp 1019 – 1028

Tanaka, J. Hirano, K., itoh, T., Nobuta, H. and Tsunoda, S., (1990), Navigation System with Map-Matching Method, Proceedings of the SAE International Congress and Exposition, pp 45 –50.