

## THE ROTATIONAL AND GRAVITATIONAL SIGNATURE OF THE DECEMBER 26, 2004 SUMATRAN EARTHQUAKE

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**Abstract.** Besides generating seismic waves, which eventually dissipate, an earthquake also generates a static displacement field everywhere within the Earth. This global displacement field rearranges the Earth's mass thereby causing the Earth's rotation and gravitational field to change. The size of this change depends upon the magnitude, focal mechanism, and location of the earthquake. The Sumatran earthquake of December 26, 2004 is the largest earthquake to have occurred since the 1960 Chilean earthquake. Using a spherical, layered Earth model, the coseismic effect of the Sumatran earthquake upon the Earth's length-of-day, polar motion, and low-degree harmonic coefficients of the gravitational field are computed. Using a model of the earthquake source that is composed of 5 subevents having a total moment-magnitude  $M_w$  of 9.3, it is found that this earthquake should have caused the length-of-day to decrease by 6.8 microseconds, the position of the Earth's generalized figure axis to shift 2.32 milliarcseconds towards 127°E longitude, the Earth's oblateness  $J_2$  to decrease by  $2.37 \times 10^{-11}$  and the Earth's pear-shapedness  $J_3$  to decrease by  $0.63 \times 10^{-11}$ . The predicted change in the length-of-day, position of the generalized figure axis, and  $J_3$  is probably not detectable by current measurement systems. But the predicted change in oblateness is perhaps detectable if other effects, such as those of the atmosphere, oceans, and continental water storage, can be adequately removed from the observations.

**Keywords:** Earth rotation, polar motion, length-of-day, gravitational field, earthquake

### 1. Introduction

The great Sumatran earthquake of December 26, 2004 is the largest earthquake to have occurred in the past 40 years and is the first magnitude 9 earthquake to have occurred during the satellite era of geophysical measurements, thereby providing an unprecedented opportunity to study the Earth's response to earthquakes from space. Given the sparseness of *in situ* measurements located near the earthquake source, space-based

measurements will be particularly important for studying the Sumatran earthquake. In this paper, the coseismic effect of the Sumatran earthquake on the Earth's rotation and gravitational field is modeled and the possibility of using space-based measurements to detect the predicted change in the Earth's rotation and gravitational field is evaluated.

Besides seismic waves and free oscillations, which eventually dissipate, an earthquake also generates a static displacement field that permanently deforms the Earth. The static displacements created by an earthquake are, of course, largest near its source and contain important information about the source mechanism of the earthquake. However, the displacements at regional to global distances are not insignificant (Press, 1965) and when integrated throughout the volume of the Earth can, at least in principle, change the Earth's rotation (Dahlen, 1971, 1973; Souriau and Cazenave, 1985; Gross, 1986; Chao et al., 1996; Chao and Gross, 2000) and global gravitational field (Chao and Gross, 1987; Gross and Chao, 1990, 2001; Sun and Okubo, 2004a, 2004b). Since the size of these changes are a function of the magnitude, location, and focal mechanism of the earthquake, their detection by space-based Earth rotation and gravitational field measurement systems would help constrain the source properties of the earthquake.

Only the initial, elastic, coseismic rotational and gravitational response of the Earth to the Sumatran earthquake is modeled here. Any viscoelastic, postseismic deformation of the Earth following the Sumatran earthquake can be expected to cause additional changes to the Earth's rotation and gravitational field that are superimposed on the initial, elastic, coseismic changes modeled here. Such postseismic changes can be as large or larger than the coseismic changes and can be expected to continue for decades after the Sumatran earthquake, but with the largest changes occurring within a few years after the earthquake. For models of viscoelastic effects on postseismic Earth rotation and local gravity changes see Dragoni et al. (1983), Sabadini et al. (1984), Soldati et al. (1998), Soldati and Spada (1999), and Wang (1999).

## 2. Model Development

The change in the Earth's rotation and gravitational field caused by the static deformation of the Earth generated by an earthquake is computed by the normal mode summation scheme described by Chao and Gross (1987) and summarized here.

### 2.1 GRAVITATIONAL FIELD

The gravitational potential  $U(\mathbf{r}_o)$  at some external field point  $\mathbf{r}_o(r_o, \theta_o, \phi_o)$  due to a body of arbitrary shape and density distribution  $\rho(\mathbf{r})$  is given by:

$$U(\mathbf{r}_o) = G \int_{V_o} \frac{\rho(\mathbf{r})}{|\mathbf{r} - \mathbf{r}_o|} dV \quad (1)$$

where  $G$  is the gravitational constant and the volume integral extends over the entire volume  $V_o$  of the body. By expanding in terms of spherical harmonics the inverse distance between the external field point and the position of some mass element located initially at  $\mathbf{r}(r, \theta, \phi)$ , the gravitational potential (1) can be written as:

$$U(\mathbf{r}_o) = 4\pi G \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{Y_{lm}^*(\theta_o, \phi_o)}{(2l+1)r_o^{l+1}} \int_{V_o} r^l Y_{lm}(\theta, \phi) \rho(r) dV \quad (2)$$

where the  $Y_{lm}(\theta, \phi)$  are the fully normalized surface spherical harmonic functions of degree  $l$  and order  $m$  and the asterisk denotes complex conjugation. However, in geodesy the gravitational potential is usually written as (e.g., Kaula, 1966):

$$U(\mathbf{r}_o) = \frac{GM}{r_o} \sum_{l=0}^{\infty} \sum_{m=0}^l \left(\frac{a}{r_o}\right)^l (C_{lm} \cos m\phi_o + S_{lm} \sin m\phi_o) \tilde{P}_{lm}(\cos\theta_o) \quad (3)$$

where  $M$  is the mass of the Earth,  $a$  is some reference radius that is less than  $r_o$  and which will be taken later to be the radius of the spherically symmetric Earth model used here, and the  $\tilde{P}_{lm}$  are the  $4\pi$ -normalized associated Legendre functions of degree  $l$  and order  $m$ . The dimensionless expansion coefficients  $C_{lm}$  and  $S_{lm}$  are known as Stokes coefficients and are the normalized multipole moments of the Earth's density field  $\rho(\mathbf{r})$ :

$$C_{lm} + i S_{lm} = \frac{N_{lm}}{Ma^l} \int_{V_o} r^l Y_{lm}(\theta, \phi) \rho(\mathbf{r}) dV \quad (4)$$

where the  $N_{lm}$  are normalization factors:

$$N_{lm} = (-1)^m \frac{2}{2l+1} \sqrt{(2-\delta_{m0})\pi} \quad (5)$$

The static displacement field generated within the Earth by an earthquake perturbs the Earth's density field which, by equation (4), causes the Stokes coefficients and hence the Earth's gravitational field to change. The change in the Stokes coefficients caused by the perturbed density field can be computed using the Lagrangian approach of following a mass element  $dM = \rho(\mathbf{r}) dV$  as it is displaced from its initial position  $\mathbf{r}$  to its perturbed position  $\mathbf{r}+\mathbf{u}$ . Assuming that the displacements  $\mathbf{u}$  caused by the earthquake are small, then the integrand in equation (4) can be expanded in a Taylor series. Keeping terms to first order yields:

$$\Delta C_{lm} + i \Delta S_{lm} = \frac{N_{lm}}{Ma^l} \int_{V_o} r^{l-1} \mathbf{u} \cdot (\hat{\mathbf{r}}l + \nabla_{\mathbf{h}}) Y_{lm}(\theta, \phi) \rho(\mathbf{r}) dV \quad (6)$$

where the hat denotes a vector of unit length,  $\nabla_{\mathbf{h}}$  is the horizontal gradient operator, and the integral extends over the initial, undeformed volume  $V_o$  of the Earth model.

## 2.2 ROTATION

When an earthquake occurs, the density distribution of the Earth is perturbed causing perturbations  $\Delta \mathbf{I}$  to the Earth's inertia tensor that, by conservation of angular momentum, causes perturbations  $\Delta \boldsymbol{\omega}$  to the angular velocity of the Earth. Changes in the equatorial components of the angular velocity of the Earth are related to the measured polar motion parameters  $p_x$  and  $p_y$  that at long periods specify the position of the rotation pole within the terrestrial reference frame. In the absence of external torques, the polar motion parameters  $p_x$  and  $p_y$  can be related to the processes causing the pole position to change by linearizing the equation, known as the Liouville equation, that expresses conservation of angular momentum within a rotating, body-fixed terrestrial reference frame (e.g., Munk and MacDonald, 1960; Lambeck, 1980; Eubanks, 1993; Gross, 2007):

$$\mathbf{p}(t) + \frac{i}{\sigma_o} \frac{d\mathbf{p}(t)}{dt} = \boldsymbol{\chi}(t) \quad (7)$$

where  $\sigma_o$  is the complex-valued frequency of the Chandler wobble,  $\mathbf{p}(t) \equiv p_x(t) - i p_y(t)$  where the negative sign accounts for the fact that by convention  $p_y(t)$  is defined to be positive toward  $90^\circ\text{W}$  longitude, and the polar motion excitation functions  $\boldsymbol{\chi}(t) \equiv \chi_x(t) + i \chi_y(t)$  are functions of the processes that are causing the terrestrial position of the rotation pole to change.

In general, the polar motion excitation functions are functions of both: (1) the changing equatorial components  $\mathbf{h}(t) \equiv h_x(t) + i h_y(t)$  of the angular momentum associated with, for example, the motion of some fluid like the atmosphere; and of (2) changes  $\mathbf{c}(t) \equiv \Delta I_{xz}(t) + i \Delta I_{yz}(t)$  in the two indicated elements of the Earth's inertia tensor (Wahr, 1982):

$$\boldsymbol{\chi}(t) = \frac{1.61}{\Omega (C - A)} \left[ \mathbf{h}(t) + \frac{\Omega \mathbf{c}(t)}{1.44} \right] \quad (8)$$

where  $\Omega$  is the mean angular velocity of the Earth ( $7.292115 \times 10^{-5}$  rad/s),  $C$  and  $A$  are the greatest and least, respectively, principal moments of inertia of the Earth with  $C-A$  being  $2.61 \times 10^{35}$  kg-m<sup>2</sup>, the factor of 1.44 accounts for the yielding of the solid Earth to imposed surface loads, and the factor of 1.61 includes the effect of core decoupling. For earthquakes, which do not load the Earth and which are assumed here to occur instantaneously and hence have no angular momentum due to motion, the polar motion excitation functions become:

$$\Delta \boldsymbol{\chi}_{\text{eq}} = \frac{1.61}{(C - A)} (\Delta I_{xz} + i \Delta I_{yz}) \quad (9)$$

The figure axis of the Earth is the principal axis of the greatest moment of inertia of the Earth. If the Earth were completely solid and rigid, then the polar motion excitation functions  $\chi_x(t)$  and  $\chi_y(t)$  would specify the location of the figure axis within the terrestrial reference frame. But because the Earth is neither completely solid nor rigid, the polar motion excitation functions are amplified by a factor of 1.61. In this case, the polar motion excitation functions can be considered to specify the location of a generalized figure axis of the Earth. The change in the location of the generalized figure axis of the Earth, that is, the change  $\Delta\chi_x$  and  $\Delta\chi_y$  in the polar motion excitation functions, due to the Sumatran earthquake is computed here.

Changes in the axial component of the angular velocity of the Earth are related to changes  $\Delta\Lambda(t)$  in the length of the day. The axial component of the linearized Liouville equation relates changes in the length-of-day to the processes that are causing it to change, namely, to the changing axial component  $h_z(t)$  of angular momentum due to motion and to changes  $\Delta I_{zz}(t)$  in the  $zz$ -element of the Earth's inertia tensor (e.g., Munk and MacDonald, 1960; Lambeck, 1980; Eubanks, 1993; Gross, 2007):

$$\Delta\Lambda(t) = \frac{\Lambda_o}{C_m \Omega} [h_z(t) + 0.756 \Omega \Delta I_{zz}(t)] \quad (10)$$

where  $\Lambda_o$  is the nominal length-of-day of 86400 seconds,  $C_m$  is the polar moment of inertia of just the Earth's crust and mantle ( $7.1242 \times 10^{37}$  kg-m<sup>2</sup>), and the factor of 0.756 accounts for the yielding of the crust and mantle to imposed surface loads. For earthquakes, which do not load the Earth and which are assumed here to occur instantaneously and hence have no angular momentum due to motion, the change in the length-of-day becomes:

$$\Delta\Lambda_{eq} = \frac{\Lambda_o}{C_m} \Delta I_{zz} \quad (11)$$

Evaluating equations (9) and (11) for the earthquake-induced changes in polar motion excitation and the length-of-day requires computing the earthquake-induced change in the Earth's inertia tensor. The inertia tensor is defined by:

$$\mathbf{I} = \int_{V_o} (r^2 \mathbf{I} - \mathbf{r} \mathbf{r}) \rho(\mathbf{r}) dV \quad (12)$$

where  $\mathbf{I}$  is the identity tensor. Like the change in the Stokes coefficients, the change in the inertia tensor caused by an earthquake can be computed using the Lagrangian approach of following a mass element  $dM = \rho(\mathbf{r}) dV$  as it is displaced from its initial position  $\mathbf{r}$  to its perturbed position  $\mathbf{r} + \mathbf{u}$ . Assuming that the displacements  $\mathbf{u}$  caused by the earthquake are small, then the integrand in equation (12) can be expanded in a Taylor series. Keeping terms to first order yields:

$$\Delta \mathbf{I} = \int_{V_o} [2(\mathbf{r} \cdot \mathbf{u}) \mathbf{I} - (\mathbf{u} \mathbf{r} + \mathbf{r} \mathbf{u})] \rho(\mathbf{r}) dV \quad (13)$$

where the integral extends over the initial, undeformed volume  $V_o$  of the Earth model.

Rather than using equation (13) to compute the change in the inertia tensor caused by the displacement field generated by an earthquake, Chao and Gross (1987) exploited the well-known relationships between the elements of the inertia tensor and the degree-2 Stokes coefficients (e.g., Heiskanen and Moritz, 1967):

$$I_{xz} = -\sqrt{5/3} Ma^2 C_{21} \quad (14)$$

$$I_{yz} = -\sqrt{5/3} Ma^2 S_{21} \quad (15)$$

$$I_{zz} = \frac{1}{3} [T - \sqrt{20} Ma^2 C_{20}] \quad (16)$$

where  $T \equiv I_{xx} + I_{yy} + I_{zz}$  is the trace of the inertia tensor. Using equations (14) and (15), equation (9) for the change in the polar motion excitation functions can be written in terms of the change in the degree-2, order-1 Stokes coefficients as:

$$\Delta \chi_{eq} = -\sqrt{\frac{5}{3}} \frac{1.61}{(C-A)} Ma^2 (\Delta C_{21} + i \Delta S_{21}) \quad (17)$$

Similarly, using equation (16), equation (11) for the change in the length-of-day can be written in terms of the change in the trace of the inertia tensor and the change in the degree-2, order-0 Stokes coefficient as:

$$\Delta \Lambda_{eq} = \frac{1}{3} \frac{\Lambda_o}{C_m} [\Delta T - \sqrt{20} Ma^2 \Delta C_{20}] \quad (18)$$

From the definition of the inertia tensor, equation (12), the trace of the inertia tensor can be shown to be:

$$T = 2 \int_{V_o} r^2 \rho(\mathbf{r}) dV \quad (19)$$

and hence, by expanding the integrand in a Taylor series and keeping terms to first order, the change in the trace of the inertia tensor caused by the displacement field generated by an earthquake is:

$$\Delta T = 4 \int_{V_o} \mathbf{r} \cdot \mathbf{u} \rho(\mathbf{r}) dV \quad (20)$$

### 2.3 EARTHQUAKE DISPLACEMENT FIELD

Evaluating equation (6) for the earthquake-induced change in the Stokes coefficients and equation (20) for the change in the trace of the inertia tensor requires modeling the static displacement field  $\mathbf{u}(\mathbf{r}, \infty)$  that is generated by the earthquake. The time-dependent displacement field  $\mathbf{u}(\mathbf{r}, t)$  is obtained by solving the equation of motion (e.g., Lapwood and Usami, 1981; Dahlen and Tromp, 1998):

$$\nabla \cdot \boldsymbol{\tau} + \mathbf{f}_g + \mathbf{f}_s(\mathbf{r}, t) = \rho(\mathbf{r}) \frac{\partial^2 \mathbf{u}(\mathbf{r}, t)}{\partial t^2} \quad (21)$$

where  $\boldsymbol{\tau}$  is the stress tensor,  $\mathbf{f}_g$  is the body force per unit volume due to self-gravitation, and  $\mathbf{f}_s(\mathbf{r}, t)$  is the body force per unit volume due to the earthquake source. The equation of motion (21) is solved here by expanding the displacement field in terms of the normal mode eigenfunctions  $\mathbf{u}_k(\mathbf{r})$ , the complete set of which forms a basis function spanning displacement space:

$$\mathbf{u}(\mathbf{r}, t) = \sum_k a_k(t) \mathbf{u}_k^*(\mathbf{r}) \quad (22)$$

where the expansion coefficients  $a_k(t)$  are given by (e.g., Lapwood and Usami, 1981, p. 185):

$$a_k(t) = \frac{1}{\omega_k} \int_{-\infty}^t \sin \omega_k(t - \tau) F_k(\tau) d\tau \quad (23)$$

with  $\omega_k$  being the eigenfrequency of the  $k$ 'th normal mode and  $F_k(t)$  being the inner product of  $\mathbf{f}_s(\mathbf{r}, t)$  with the  $k$ 'th normal mode eigenfunction:

$$F_k(t) = \int_{V_o} \mathbf{f}_s(\mathbf{r}, t) \cdot \mathbf{u}_k(\mathbf{r}) dV \quad (24)$$

For spatial dimensions much larger than the dimensions of the earthquake's source volume, and for time spans much longer than the earthquake's duration, the earthquake source can be conveniently represented as a point source with a step function time history. In this case,  $\mathbf{f}_s(\mathbf{r}, t)$  can be written in terms of the moment tensor  $\mathbf{M}$  as (e.g., Aki and Richards, 1980, p. 351):

$$\mathbf{f}_s(\mathbf{r}, t) = -\mathbf{M} \cdot \nabla \delta(\mathbf{r} - \mathbf{r}_s) H(t - t_s) \quad (25)$$

where  $\delta(\mathbf{r} - \mathbf{r}_s)$  is the Dirac delta function,  $H(t - t_s)$  is the Heaviside step function,  $\mathbf{r}_s$  is the location of the centroid of the earthquake's source volume, and  $t_s$  is the centroid time of the earthquake.

The normal mode expansion coefficients in the static limit  $a_k(\infty)$  can now be obtained by combining equations (23–25) and performing the respective integrations (e.g., Gilbert, 1970):

$$a_k(\infty) = \frac{1}{\omega_k^2} \mathbf{M} : \boldsymbol{\epsilon}_k(\mathbf{r}_s) \quad (26)$$

where  $\boldsymbol{\epsilon}_k(\mathbf{r}_s)$  is the strain tensor of the  $k$ 'th normal mode eigenfunction evaluated at the centroid of the earthquake's source volume:

$$\boldsymbol{\epsilon}_k(\mathbf{r}_s) = \frac{1}{2} \left[ \nabla \mathbf{u}_k(\mathbf{r}_s) + (\nabla \mathbf{u}_k(\mathbf{r}_s))^\top \right] \quad (27)$$

with the superscript  $\top$  denoting transpose. Equations (22) and (26–27) thus give the modeled static displacement field needed in equations (6) and (20) to compute the earthquake-induced changes to the Stokes coefficients and to the trace of the inertia tensor given the normal mode eigenfunctions  $\mathbf{u}_k(\mathbf{r})$  and eigenvalues  $\omega_k$  of some Earth model and the moment tensor representation of the earthquake's source.

In general, a spherically symmetric, non-rotating, elastic, isotropic (SNREI) Earth model has both toroidal and spheroidal normal modes. However, only the spheroidal modes need to be considered here since the displacements associated with the toroidal modes have no component in the radial direction and hence these modes cannot affect the rotation or gravitational field of a SNREI Earth model. The eigenfunctions  ${}_n\sigma_l^m(\mathbf{r})$  of the spheroidal normal modes can be written in terms of scalar functions of radius  ${}_nU_l(r)$  and  ${}_nV_l(r)$  as (e.g., Gilbert and Dziewonski, 1975):

$${}_n\sigma_l^m(\mathbf{r}) = {}_nU_l(r) Y_{lm}(\theta, \phi) \hat{\mathbf{r}} + {}_nV_l(r) \nabla_{\mathbf{h}} Y_{lm}(\theta, \phi) \quad (28)$$

where  $n$  is the radial overtone number. Since the Earth model is spherically symmetric, the scalar eigenfunctions  ${}_nU_l(r)$  and  ${}_nV_l(r)$  do not depend on order  $m$ . The final expression for the modeled change to the Stokes coefficients caused by the coseismic effects of an earthquake is obtained by combining equations (6), (22), and (26–28):

$$\Delta C_{lm} + i \Delta S_{lm} = \frac{N_{lm}}{Ma^l} \sum_{n=0}^{\infty} {}_nF_l \left[ \mathbf{M} : {}_n\boldsymbol{\epsilon}_l^m(\mathbf{r}_s) \right] \quad (29)$$

where:

$${}_nF_l = \frac{l}{({}_n\omega_l)^2} \int_0^a \rho(r) r^{l+1} [{}_nU_l(r) + (l+1) {}_nV_l(r)] dr \quad (30)$$

The final expression for the modeled change to the trace of the inertia tensor caused by the coseismic effects of an earthquake is obtained by combining equations (20), (22), and (26–28):

$$\Delta T = \sum_{n=0}^{\infty} {}_n G_0 [\mathbf{M} : {}_n \boldsymbol{\epsilon}_0^0(\mathbf{r}_s)] \quad (31)$$

where:

$${}_n G_0 = \frac{\sqrt{64\pi}}{({}_n \omega_0)^2} \int_0^a \rho(r) r^3 {}_n U_0(r) dr \quad (32)$$

Since the seismic moment tensor  $\mathbf{M}$  is linearly proportional to the scalar moment of the earthquake (e.g., Aki and Richards, 1980), it is seen from equation (29) that the change to the Stokes coefficients and from equation (31) that the change to the trace of the inertia tensor are also linearly proportional to the scalar moment of the earthquake. Thus, apart from differences in focal mechanism and source location, the largest earthquakes can be expected to have the largest effects on the Earth's rotation and gravitational field.

### 3. 2004 Sumatran Earthquake

The Sumatran earthquake of December 26, 2004 is the largest earthquake to have occurred since the 1960 Chilean earthquake. To model its effect on the Earth's rotation and gravitational field, the Harvard centroid-moment tensor (CMT; Dziewonski et al, 1987, 1990) solution for its source properties is used here. The standard Harvard CMT solution for the 2004 Sumatran earthquake placed the source centroid at about 3°N latitude and assigned a seismic moment of  $3.95 \times 10^{29}$  dyne-cm ( $M_w = 9.0$ ) to it. However, free oscillation data (Stein and Okal, 2005) placed the source centroid at about 7°N latitude and assigned a seismic moment of  $10.0 \times 10^{29}$  dyne-cm ( $M_w = 9.3$ ) to it. Extending the standard Harvard CMT solution by using 5 point sources to model a propagating slip pulse, Tsai et al. (2005) found a total seismic moment of  $11.66 \times 10^{29}$  dyne-cm ( $M_w = 9.3$ ) for it with the locations of the point sources ranging from about 3°N to 13°N latitude.

The SNREI Earth model used here is the transversely isotropic version of the Preliminary Reference Earth Model (PREM; Dziewonski and Anderson, 1981). The normal mode eigenfunctions  ${}_n U_l(r)$  and  ${}_n V_l(r)$  and eigenvalues  ${}_n \omega_l$  of PREM were computed using the MINOS computer code provided to us by G. Masters (personal communication, 1999). The spherical harmonics  $Y_{lm}(\theta, \phi)$  that are needed in equation (27) when computing the strain tensor of the  $k$ 'th normal mode eigenfunction (see Chao and

TABLE I  
Modeled effect of the 2004 Sumatran and other great earthquakes on the Earth's rotation

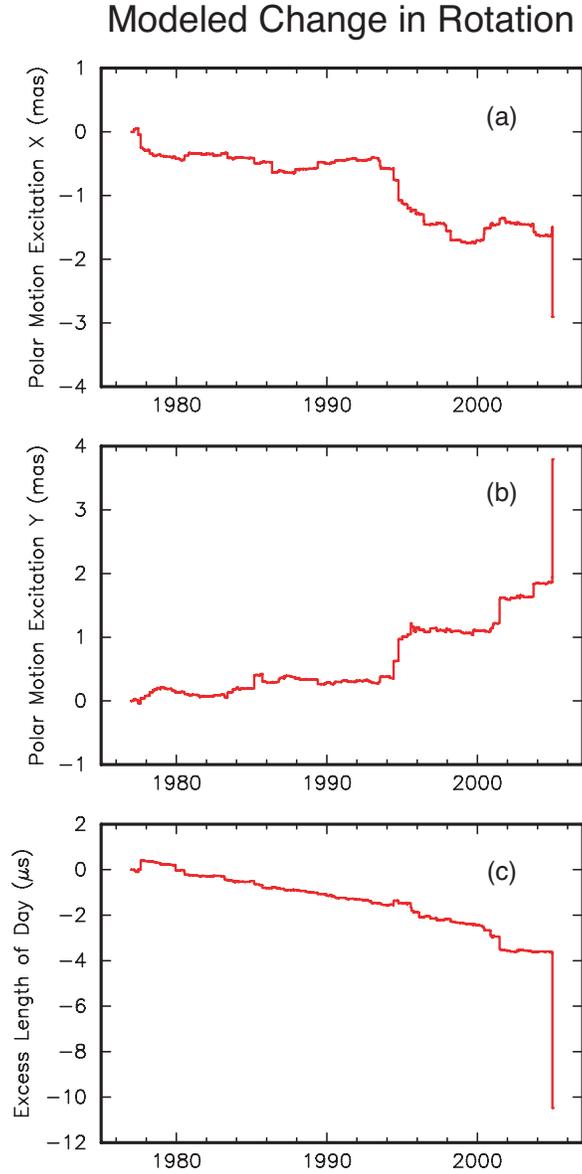
<i>Source Description</i>	$\Delta\Lambda$ ( $\mu\text{s}$ )	$\Delta\chi_x$ ( <i>mas</i> )	$\Delta\chi_y$ ( <i>mas</i> )
<i>2004 Sumatran earthquake</i>			
$M_w = 9.0$ ; 3°N (standard Harvard CMT solution)	-2.702	-0.673	0.500
$M_w = 9.3$ ; 3°N (larger moment)	-6.841	-1.703	1.266
$M_w = 9.3$ ; 7°N (larger moment, centroid moved N)	-6.594	-1.799	2.195
$M_w = 9.3$ ; 3°N–13°N (Tsai et al., 2005)	-6.766	-1.407	1.846
<i>Other great earthquakes</i>			
1960 Chilean (Chao and Gross, 1987)	-8.40	-9.53	20.45
1964 Alaskan (Chao and Gross, 1987)	6.79	-7.11	-2.31
<i>Approximate Earth rotation measurement uncertainty (<math>1\sigma</math>)</i>	20	5	5

Gross, 1987, eq. 34) were computed using the XFCN2 computer code of Masters and Richards-Dinger (1998).

### 3.1 ROTATION

Table I gives the modeled change in the length-of-day and polar motion excitation functions that should have been caused by the 2004 Sumatran earthquake. Starting with the standard Harvard CMT solution for the source, the length-of-day should have decreased by 2.7 microseconds ( $\mu\text{s}$ ). If the focal mechanism and location of the source centroid is kept the same while the moment-magnitude is increased to 9.3, to agree with that inferred from free oscillation data, then the length-of-day should have decreased by 6.8  $\mu\text{s}$ . Moving the source centroid from 3°N to 7°N while keeping the focal mechanism and moment-magnitude the same at 9.3 has only a small impact on the modeled change in the length-of-day, as does using the distributed source model of Tsai et al. (2005). Since length-of-day changes can currently be determined with an accuracy of about 20  $\mu\text{s}$ , it is seen that the modeled change in the length-of-day caused by the 2004 Sumatran earthquake is about a factor of 3 smaller than the uncertainty in length-of-day measurements.

The generalized figure axis should have shifted by 0.84 milliarcseconds (*mas*) towards 143°E longitude using the standard Harvard CMT solution for the source. Increasing the moment-magnitude to 9.3 while keeping the focal mechanism and the location of the source centroid the same increases the amplitude of the shift to 2.12 *mas* but keeps the direction the same (towards 143°E longitude). Moving the source centroid from 3°N to 7°N while keeping the focal mechanism and moment-magnitude the same at 9.3 would cause the generalized figure axis to shift by 2.84 *mas* towards 129°E longitude. Using the 5 subevent model of Tsai et al. (2005) for the source, the generalized figure axis of the Earth is predicted to shift by 2.32 *mas* towards 127°E longitude. Since polar motion excitation functions can currently be determined with an accuracy of about 5 *mas*, it is seen that, like the modeled length-of-day change, the modeled change in the polar



*Figure 1.* Modeled cumulative change in (a) the  $x$ -component of polar motion excitation, (b) the  $y$ -component of polar motion excitation, and (c) the length-of-day caused by 22,369 earthquakes having moment-magnitude  $M_w$  greater than about 5.5 that occurred during 1977–2004. The modeled effect of the 2004 Sumatran earthquake is the large change evident at the end of the series.

motion excitation functions is also about a factor of 3 smaller than the measurement uncertainty.

To place these results in context, Figure 1 shows the modeled cumulative change in the length-of-day and polar motion excitation functions caused by the 22,369 earthquakes in the Harvard CMT catalog that occurred during 1977–2004. The results shown in Figure 1 for the 2004 Sumatran earthquake were computed using the distributed source model of Tsai et al. (2005); results for the other 22,368 earthquakes were computed using the standard Harvard CMT solution for the source. As can be seen, the modeled changes caused by the 2004 Sumatran earthquake are substantially larger than those caused by any of the other earthquakes that occurred during 1977–2004.

To compare the results for the 2004 Sumatran earthquake with those predicted for other great earthquakes, Table I also gives the modeled change in the Earth’s rotation for the 1960 Chilean and 1964 Alaskan earthquakes that were determined by Chao and Gross (1987). Even though the 1964 Alaskan earthquake was smaller than the 2004 Sumatran earthquake, with an estimated seismic moment of  $7.5 \times 10^{29}$  dyne-cm (Kanamori, 1970), its effect on the length-of-day is predicted to be about the same as that of the Sumatran earthquake, although of opposite sign, and its effect on the position of the generalized figure axis is predicted to be about 3 times larger, shifting it by 7.5 mas towards  $198^\circ\text{E}$  longitude. A shift in the position of the generalized figure axis of this size should be detectable by present-day measurement systems, as should the shift of 22.6 mas towards  $115^\circ\text{E}$  longitude predicted to be caused by the 1960 Chilean earthquake. But the predicted change in the length-of-day caused by the 1960 Chilean and 1964 Alaskan earthquakes, like that of the 2004 Sumatran earthquake, would probably not be detectable by present-day measurement systems. The result that the 1964 Alaskan earthquake should have had a larger effect than the 2004 Sumatran earthquake on the position of the generalized figure axis even though it has a smaller magnitude illustrates the importance of the earthquake’s location and focal mechanism, in addition to its magnitude, on determining the effect that the earthquake has on the Earth’s rotation.

### 3.2 GRAVITATIONAL FIELD

Table II gives the modeled change in selected low-degree zonal harmonic coefficients of the Earth’s gravitational field that should have been caused by the 2004 Sumatran earthquake. The tabulated values are given in terms of the change in the dimensionless zonal  $J_l$  coefficients defined by:

$$\Delta J_l = -\sqrt{2l+1} \Delta C_{l0} \quad (33)$$

where  $\Delta C_{l0}$  is the change in the zonal Stokes coefficient. Starting with the standard Harvard CMT solution for the source,  $J_2$  should have decreased by  $0.9 \times 10^{-11}$ . If the focal mechanism and location of the source centroid is kept the same while the moment-magnitude is increased to 9.3, to agree with that inferred from free oscillation data, then  $J_2$  would decrease by  $2.3 \times 10^{-11}$ . Moving the source centroid from  $3^\circ\text{N}$  to  $7^\circ\text{N}$  while keeping the focal mechanism and moment-magnitude the same at 9.3 has only a small

TABLE II  
Modeled effect of the 2004 Sumatran and other great earthquakes on the Earth's gravitational field

<i>Source Description</i>	$\Delta J_2$ ( $10^{-11}$ )	$\Delta J_3$ ( $10^{-11}$ )	$\Delta J_4$ ( $10^{-11}$ )	$\Delta J_5$ ( $10^{-11}$ )
<i>2004 Sumatran earthquake</i>				
$M_w = 9.0$ ; 3°N (standard CMT solution)	-0.906	-0.206	0.093	0.138
$M_w = 9.3$ ; 3°N (larger moment)	-2.293	-0.521	0.234	0.349
$M_w = 9.3$ ; 7°N (larger moment, centroid to N)	-2.166	-0.748	0.096	0.279
$M_w = 9.3$ ; 3°N–13°N (Tsai et al., 2005)	-2.368	-0.628	0.250	0.312
<i>Other great earthquakes</i>				
1960 Chilean (Chao and Gross, 1987)	-0.83	3.29	-1.89	3.64
1964 Alaskan (Chao and Gross, 1987)	5.25	2.35	1.40	1.62
<i>Approximate measurement uncertainty (<math>1\sigma</math>)</i>	1.3	1.6	4.9	

impact on the modeled change in  $J_2$ , as does using the distributed source model of Tsai et al. (2005). Since changes in  $J_2$  can currently be determined from satellite laser ranging observations with an accuracy of about  $1.3 \times 10^{-11}$  (C. Cox, personal communication, 2005), it is seen that the modeled change in  $J_2$  caused by the 2004 Sumatran earthquake is about a factor of 2 larger than the uncertainty in the measurements of  $J_2$ . Thus, it may be possible to detect the change in  $J_2$  caused by the 2004 Sumatran earthquake in satellite laser ranging measurements. However, from Table II, the modeled change in the other low-degree zonal coefficients of the Earth's gravitational field is seen to be much less than the uncertainty in their measurements.

Figure 2 shows the modeled cumulative change in the low-degree zonal harmonic coefficients of the Earth's gravitational field caused by the 22,369 earthquakes in the Harvard CMT catalog that occurred during 1977–2004. The results shown in Figure 2 for the 2004 Sumatran earthquake were computed using the distributed source model of Tsai et al. (2005); results for the other 22,368 earthquakes were computed using the standard Harvard CMT solution for the source. Like its modeled effect on the Earth's rotation, the change in the low-degree zonal harmonics of the Earth's gravitational field caused by the 2004 Sumatran earthquake is substantially larger than the change caused by any of the other earthquakes.

Table II also gives the modeled change in the low-degree zonal harmonic coefficients of the Earth's gravitational field for the 1960 Chilean and 1964 Alaskan earthquakes that were determined by Chao and Gross (1987). Like the Earth rotation changes, the modeled changes in the gravitational field coefficients are larger for the 1964 Alaskan earthquake than they are for the 2004 Sumatran earthquake. In fact, the predicted  $J_2$  and  $J_3$  changes are large enough that they should be detectable in present-day satellite laser ranging measurements, as should the predicted  $J_3$  change caused by the 1960 Chilean earthquake. Because of differences in location and focal mechanism, the predicted  $J_2$  change caused by the 1960 Chilean earthquake is much less than that predicted for either the 1964 Alaskan or 2004 Sumatran earthquakes, even though the 1960 Chilean earthquake was larger.

## Modeled Change in Gravitational Field

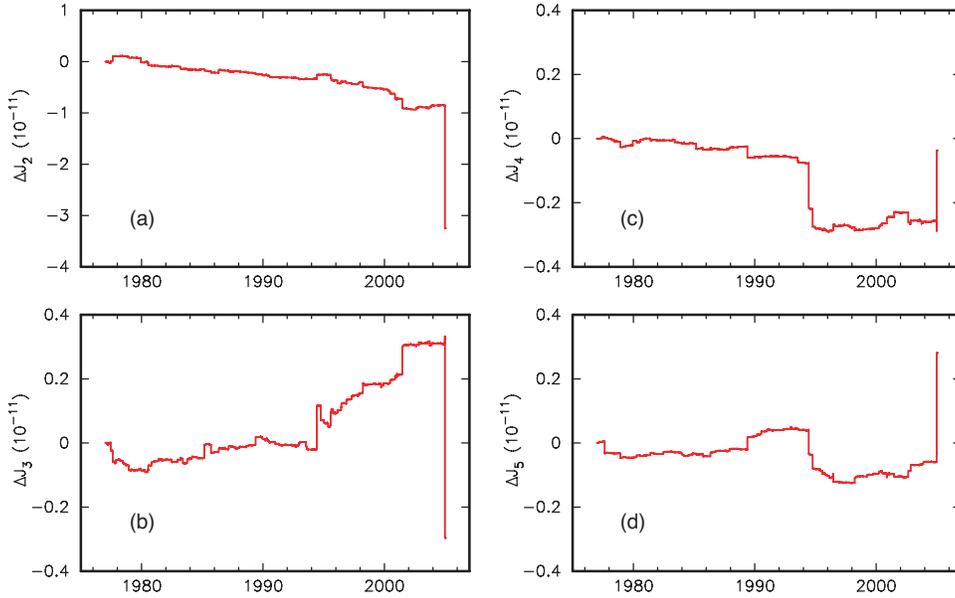


Figure 2. Modeled cumulative change in the (a)  $J_2$ , (b)  $J_3$ , (c)  $J_4$ , and (d)  $J_5$  coefficients of the Earth's gravitational field caused by 22,369 earthquakes having moment-magnitude  $M_w$  greater than about 5.5 that occurred during 1977–2004. The modeled effect of the 2004 Sumatran earthquake is the large change evident at the end of the series.

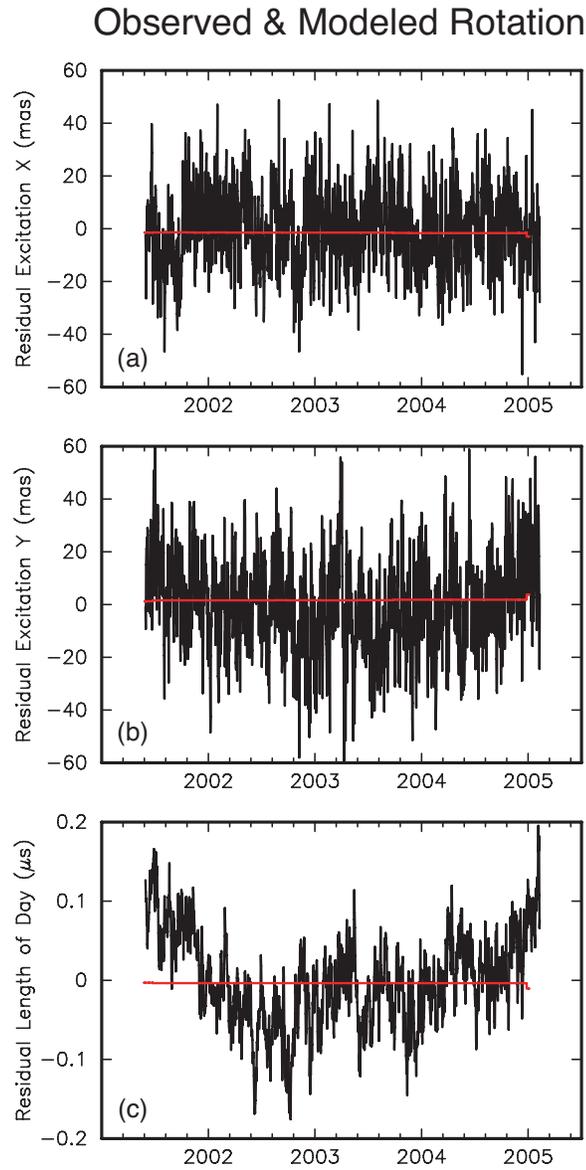
### 4. Comparison of Modeled and Observed Rotation Changes

While the modeled change in the Earth's rotation that should have been caused by the 2004 Sumatran earthquake is less than the uncertainty in present-day measurements of the Earth's rotation, it is still worth examining the measurements to see if an earthquake-induced signal is present. The observed polar motion excitation and length-of-day series used here is derived from the operational version of the combined Earth rotation series generated at the Jet Propulsion Laboratory (JPL) in support of interplanetary spacecraft tracking and navigation (Gross et al., 1998). This observed series is a combination of Earth rotation measurements taken by the techniques of lunar and satellite laser ranging, very long baseline interferometry, and the global positioning system. Besides estimating Universal Time (UT1) and polar motion, the Kalman filter used to combine the Earth rotation measurements also estimates their time rates-of-change, including the length-of-day. The polar motion excitation functions have been determined from the operational polar motion and polar motion rate estimates using equation (7) where the period  $T_o$  of the Chandler wobble was assumed to be 433.0 days and its quality factor  $Q_o$  was

assumed to be 179 (Wilson and Vicente, 1990). The resulting observed polar motion excitation and length-of-day series spans May 30, 2001 to February 9, 2005 at daily intervals.

Prior to searching the observed Earth rotation series for an earthquake-induced signal, other sources of polar motion excitation and length-of-day variations that can be accurately modeled should be removed from the observations. The model of Yoder et al. (1981) for the long-period solid Earth tides and the model of Kantha et al. (1998) for the long-period ocean tides have been used to remove these effects from the length-of-day observations. Atmospheric effects were removed using an atmospheric angular momentum (AAM) series derived from the products of the National Centers for Environmental Prediction (NCEP) operational analysis system that was obtained from the International Earth Rotation and Reference Systems Service (IERS) Special Bureau for the Atmosphere (SBA; Salstein et al., 1993). Both the angular momentum of the winds and the angular momentum associated with surface pressure variations, computed by assuming that the oceans respond as an inverted barometer to the imposed surface pressure variations, were used. Oceanic effects were removed using an oceanic angular momentum (OAM) series that was obtained from the IERS Special Bureau for the Oceans (SBO). The particular OAM series used was derived from the products of a data assimilative ocean model, designated kf049f, that was run at JPL as part of their participation in the Estimating the Circulation and Climate of the Ocean (ECCO) consortium (Stammer et al., 2002). Both the angular momentum of the currents and the angular momentum associated with bottom pressure variations were used. Since the ocean model was not forced by surface pressure variations, the response of the oceans to surface pressure variations is assumed to be like an inverted barometer and has been included in the AAM series.

Figure 3 shows the observed polar motion excitation and length-of-day series (black curves) from which tidal, atmospheric, and oceanic effects have been removed as described above. A residual seasonal signal has also been removed by a simultaneous unweighted least-squares fit for a mean, trend, and periodic signals at the annual, semiannual, and terannual (3 cycles/year) frequencies. Superimposed on the observed black curves are the modeled series shown in Figure 1 (red curves). The predicted effect of the 2004 Sumatran earthquake is the small change evident at the end of the modeled red curves. As can be seen, the scatter in the residual observations is much larger than the modeled signal and there is no evidence for a step-like change in the observations at the time of the earthquake; consequently, there is no evidence for an earthquake-induced signal in the Earth rotation observations, either in polar motion excitation or in length-of-day.



*Figure 3.* Observed and modeled changes in (a) the x-component of polar motion excitation, (b) the y-component of polar motion excitation, and (c) the length-of-day. The observed series (black curves) have had atmospheric, oceanic, and seasonal effects removed. Tidal effects have also been removed from the observed length-of-day series. The modeled series (red curves) are those shown in Figure 1. The predicted effect of the 2004 Sumatran earthquake is the small change evident at the end of the modeled series.

## 5. Discussion and Summary

The approach used by Chao and Gross (1987) to model the elastic, coseismic effect of earthquakes on the Earth's rotation and gravitational field has been summarized here and applied to the great Sumatran earthquake of December 26, 2004. The expected change in the Earth's rotation was found to be about 3 times smaller than the uncertainty in present-day observations of the Earth's rotation. Not surprisingly, an examination of the observations from which tidal, atmospheric, and oceanic effects had been removed showed no evidence for a change in the Earth's rotation having the expected step-like signature at the epoch of the earthquake.

The expected change in the low-degree zonal harmonic coefficients of the Earth's gravitational field was also smaller than the uncertainty in present-day observations of the Earth's gravitational field determined from satellite laser ranging measurements, except for the degree-2 coefficient. The expected change in this coefficient was about twice as large as the measurement uncertainty and so it may be possible to detect a step-like change at the epoch of the earthquake in satellite laser ranging determinations of this coefficient if other effects, such as those of the atmosphere, oceans, and continental water storage, can be adequately removed from the observations.

The modeled results presented here for the change in the Earth's rotation and gravitational field caused by the 2004 Sumatran earthquake using the standard Harvard CMT solution for its source differ by up to about 6% from those reported by Chao and Gross (2005). This difference is due to differences in the Earth model used in the two studies. Here, the transversely isotropic version of PREM was used, whereas Chao and Gross (2005) used the fully isotropic version of PREM. Experiments using Earth model 1066B of Gilbert and Dziewonski (1975) produced differences of up to 10%. Thus, differences in realistic Earth models can produce results that differ by up to about 10% in the modeled earthquake-induced changes in the Earth's rotation and low-degree harmonic coefficients of the Earth's gravitational field.

Differences in magnitude, focal mechanism and location also cause differences in the modeled effect of earthquakes on the Earth's rotation and gravitational field. Detecting the effect of earthquakes in space-based measurements of the Earth's rotation and gravitational field would allow such measurements to be used to constrain the source properties of the earthquake. This would be particularly important for great earthquakes located in remote regions of the Earth, like the 2004 Sumatran earthquake, where there may be few *in situ* instruments located near the source that survive the earthquake. The joint United States–German Gravity Recovery And Climate Experiment (GRACE) satellite mission is currently measuring changes in the Earth's gravitational field from space. A number of research groups are actively examining these measurements for changes caused by the 2004 Sumatran earthquake. If such a change is detected, then that measurement in concert with models of the change, such as that presented here, could be used to help settle the controversy about the magnitude and source properties of the 2004

Sumatran earthquake and would lead to greater understanding of the physics of such highly destructive great subduction zone earthquakes.

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