JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 110, B05406, doi:10.1029/2004JB003497, 2005

Uncertainty of absolute gravity measurements

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Received 21 October 2004; revised 28 January 2005; accepted 3 March 2005; published 24 May 2005.

[1] A total of 96 absolute gravity (AG) measurements at the Membach station and 221 at the Proudman Oceanographic Laboratory (POL) is analyzed for noise content. The lengths of the series were around 10 years (POL) and 8 years (Membach). First the noise at frequencies lower than 1 cpd is studied. This noise consists in setup-dependent offsets and geophysical colored sources. The setup white noise is estimated using continuous relative superconducting gravity (SG) measurements at Membach. The colored environmental noise affecting both AG and SG is estimated using the maximum likelihood estimation technique to fit two types of stochastic models to the SG time series, power law noise, and first-order Gauss Markov (FOGM) noise. We estimate the noise amplitudes of a white noise process plus power law model while simultaneously solving for the spectral index and the noise amplitudes of a white noise process plus FOGM noise model is also estimated. The gravity rate of change and the associated uncertainties as a function of the noise structure are then computed. At frequencies higher than 1 cpd, a time-varying white noise component usually dominates AG time series. Finally, the POL and Membach experiments are applied to estimate the uncertainties for AG campaigns repeated once or twice a year to monitor crustal deformation. Such repeated AG measurements should allow one to constrain gravity rate of change with an uncertainty of 1 nm s^{-2} yr^{-1} (or 0.5 mm yr^{-1}) after 14 or 24 years, depending on the noise model. Therefore long-term measurements using absolute gravimeters are appropriate for monitoring slow vertical tectonic deformation.

Citation: Van Camp, M., S. D. P. Williams, and O. Francis (2005), Uncertainty of absolute gravity measurements, *J. Geophys. Res.*, *110*, B05406, doi:10.1029/2004JB003497.

1. Introduction

[2] To observe secular land movements of the order of a few millimeters per year, a very precise instrument with long-term stability is required. This can be achieved using absolute gravimeters [*Williams et al.*, 2001]. Vertical land movements would modify the gravity at a rate of about −10 nm s^{-2} (1 μGal) for 5 mm of uplift [*Ekman and Mäkinen*, 1996]. The deformation rate is given by the slope of a linear trend fitted to the repeated time series.

[3] Usually, the regression estimator used is some form of least squares adjustment and the measurement errors are assumed to be normal (Gaussian) and statistically uncorrelated from one another (white noise). However, many geodetic data sets have now provided evidence for error sources that introduce large temporal correlations into the data [*Agnew*, 1992]. One common statistical model for many types of geophysical signal (which may contribute to the noise) may be described as a power law process [*Mandelbrot*, 1983; *Agnew*, 1992]. The stochastic process is such that its power spectrum has the form

$$P_x(f) = P_0\left(\frac{f}{f_0}\right)^{\kappa} \qquad (1)$$

where f is the spatial or temporal frequency, P_0 and f_0 are normalizing constants, and κ is the spectral index [*Mandelbrot and Van Ness*, 1968]. For $\kappa = 0$, we have classical white noise and for $\kappa \neq 0$ we refer to colored noise. If $-3 < \kappa < -1$, we have "fractional Brownian motion"; if $\kappa = -1$ we have "flicker" or "pink" noise and, if $\kappa = -2$, we have random walk. When the random walk spectrum flattens toward low frequencies, we have the first-order Gauss Markov (FOGM) noise, such that the power spectrum has the form

$$P_x(f) = \sigma_{rw}^2 \frac{1}{\beta^2 + 4\pi^2 f^2} \qquad (2)$$

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where β is approximately equivalent to the cross over frequency and σ_{rw} is the scaling for the random walk part of the model.

[4] The power law process has been observed in geodetic time series such as continuously recording strain meters [*Agnew*, 1992], GPS [*Zhang et al.*, 1997; *Mao et al.*, 1999; *Williams et al.*, 2004] and sea level changes [*Harrison*, 2002]. Accounting for the type of noise is very important when estimating the related uncertainties but does not influence the slope estimate significantly [*Williams*, 2003].

[5] The spectrum of a time series of measurements can be modeled as the sum of a white noise and colored noise. The frequency at which the colored and white noises are equal is the crossover frequency. For an ordinary globally referenced GPS position time series, the length of data series required to detect the Random Walk noise in the power spectrum is about 8 years in the best cases [*Johnson and Agnew*, 2000]. We show here that this is reduced to less than 1 week combining SG and AG measurements.

[6] The expected power spectra of gravity measurements, based on previous studies of seismic noise, were discussed by *Lambert et al.* [1995]. In particular, they supposed power law noise toward low frequencies. They already pointed out the need to combine absolute gravity and superconducting gravity measurements to achieve the optimum noise characteristics of both instruments. With this aim in view they realized that more work had to be done in characterizing the spectra of actual AG and SG data. *Francis et al.* [1998] published a first noise estimate for the FG5 absolute gravimeter. Because of the lack of available data, the errors were assumed statistically uncorrelated (white noise process). This underestimated the noise level at frequencies lower than about 0.1–1 cycle per day (cpd). A spectral comparison of actual AG and SG time series was presented by *Crossley et al.* [2001]. Three years of real data were used to compare with the *Lambert et al.* [1995] theoretical spectrum and negligible differences were found for periods longer than 1 day. The power law noise and the AG instrumental white noise at periods shorter than one day were both obvious in the spectrum. However, neither the influence of the power law noise on the uncertainties of estimated deformation rates or the AG setup noise were discussed.

[7] The aim of this paper is to present a more detailed analysis of the noise affecting absolute gravity measurements. This is done by comparing superconducting gravimeter (SG) and AG data individually and by applying the statistical method presented by *Zhang et al.* [1997] and *Williams* [2003]. Because of mechanical wear, absolute gravimeters are not well suited for continuous measurements lasting longer than a few days or a few weeks. The SG provides continuous data between episodic AG measurements over many years and allows easy computation of the geophysical noise spectrum from 0.1 Hz (seismic surface waves and Earth's free oscillations) to less than 1 cycle per year (Chandler wobble) [*Crossley et al.*, 1999]. SG data, however, have the disadvantage of drifting with time, which can be evaluated and removed by performing regular side-by-side AG measurement [*Francis et al.*, 2004a]. This is done assuming the AG setup-dependent offsets consist of a Gaussian white noise that should not influence the measurement of the long-term geophysical trend. Then, comparing drift-free SG data with AG time series provides information on the geophysical noise affecting AG at low frequencies (except DC) and on the uncertainties due to the setup of the AG instrument.

[8] With an improved understanding of the noise, we estimate the ability of AG to monitor vertical crustal deformations. In particular, we provide more realistic uncertainties of the geophysical trend observed at the Membach station [*Francis et al.*, 2004a]. Moreover, using this experience, we evaluate the uncertainties that can be expected when carrying out repeated AG campaigns at other stations. This study can be very useful considering the repeated AG measurement campaigns undertaken since the 1990s to measure crustal deformation (intraplate and interplate tectonic deformation [*Zerbini et al.*, 2001; *Van Camp et al.*, 2002; *Hinderer et al.*, 2003], postglacial rebound [*Lambert et al.*, 2001; *Williams et al.*, 2001], anthropogenic subsidence [*Van Camp*, 2003a] or ice-mass and water-mass changes [*van Dam et al.*, 2000]). Finally we provide advice for measuring Absolute Gravity in a noisy environment.

2. The Data

[9] The most accurate, and in fact the only commercially available absolute gravimeter, is the FG5 from Micro-g Solutions [*Niebauer et al.*, 1995]. A test mass is repeatedly dropped and its position is measured as a function of time. In routine operation, the drops are repeated every 10 s, 100 to 200 times per hour. The average of 100–200 drops is a "set" and measurements usually consist of one set per hour. Recording a set takes 17 to 34 min; in other words, there is a gap of 26 to 43 min in the time series of 1 data per 10 s. The average of several sets provides a "gravity value". The instrumental accuracy of the FG5 is about 10–20 nm s^{-2} as reported by the manufacturer [*Niebauer et al.*, 1995]. However, because we cannot model the environmental effects perfectly, the precision in practice is frequency-dependent, as shown in this paper.

[10] The fundamental component of a SG consists of a hollow superconducting sphere that levitates in a persistent magnetic field [*Goodkind*, 1999]. An incremental change in gravity induces a vertical displacement of the sphere. The GWR SG provides relative gravity measurements and the most common mode of operation is continuous at a fixed location. Being a relative meter, SGs do not provide the gravity and the SG C021 of Membach is calibrated by operating an absolute gravimeter side by side. This method of calibration allows for a precision in the calibration factor better than 0.1% [*Francis et al.*, 1998].

[11] The AG data processing was done following the standard method applied at the BIPM [*Vitushkin et al.*, 2002; *Francis and van Dam*, 2003]. For a description of SG data processing, see, e.g., *Francis et al.* [2004a]. Our study used 96 AG values recorded by the FG5 202 and SG data over a period of 8 years at the Membach station (Belgium) [*Francis et al.*, 2004a] and more than 200 AG values taken by the FG5 103 at the Proudman Oceanographic Laboratory (POL, Bidston, UK) since 1994.

3. Estimating the Uncertainties

[12] First we compare AG and SG data at the Membach station, providing an independent way to estimate the precision of AG measurements. Removing AG data from

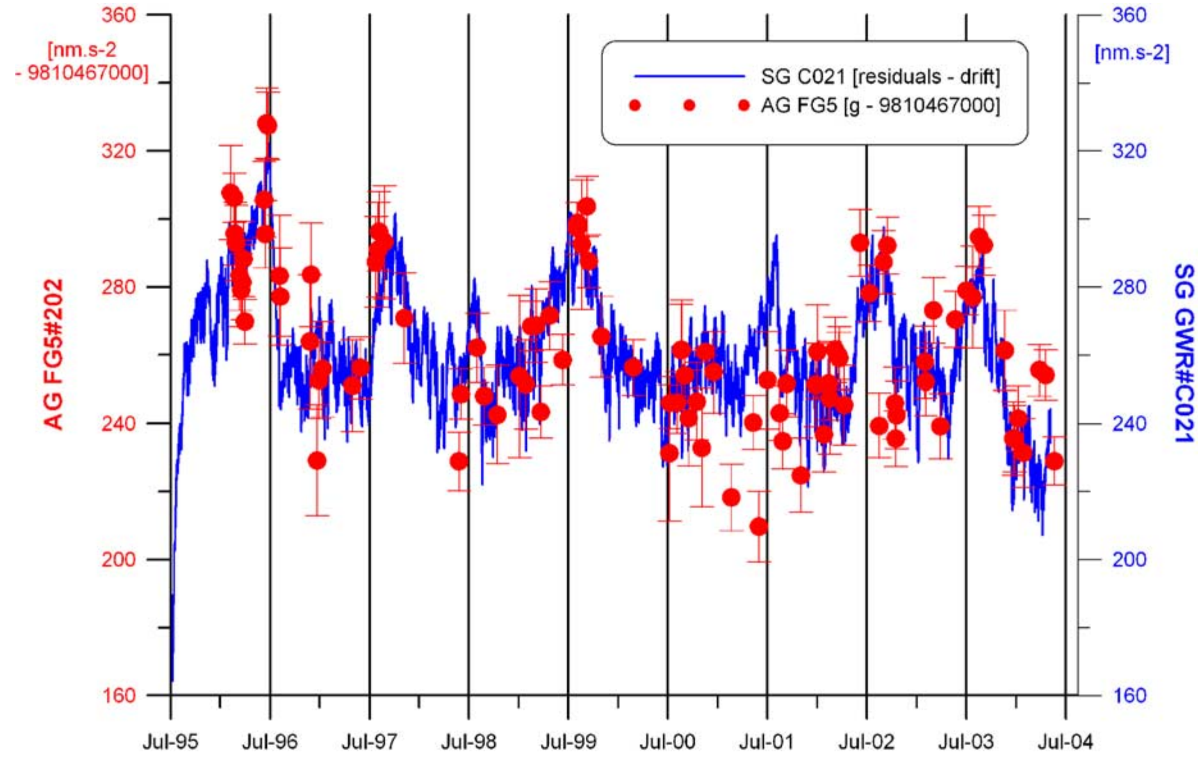

Figure 1. Comparison between gravity measurements of the drift-corrected SG C-021 and the AG FG5-202 at the Membach station. Each AG gravity value represents the average of 2000 to 20,000 drops, equivalent to 1 to 8 days. The Earth tides, ocean loading and atmospheric effects, and polar motion have been removed. The SG C-021 instrumental drift of 43 nm s^{-2} yr^{-1} was evaluated by fitting a first-order polynomial on the difference between the SG and the AG gravity data. The initial exponential decrease from August to December 1995 is due to the SG C-021 setup. Then from March 1996 to June 2004, a geophysical trend, mainly linear, can be observed in these corrected SG residuals (see Table 1 for values and uncertainties).

SG allows the SG instrumental drift to be removed. The higher precision at high frequencies of the continuous SG allows us to evaluate the noise due to the setup of the FG5, and therefore provide a better estimate of the error bars. This noise results from instrumental setup-dependent offsets and can be due for example to poor alignment of the instrument, errors in height measurement, slight perturbations due to transportation or different instrument-floor couplings. We use the combined AG and continuous SG data to evaluate the crossover frequency at which the AG instrumental white and geophysical colored noise sources have equal power. The crossover frequency is between 0.1 and 1 cycle per day (cpd). As there are no significant differences in the AG and SG at frequencies lower than 0.1–1 cpd, this colored noise affects the AG measurements identically. We then apply the methods proposed by *Williams* [2003] to compute the gravity rate of change and the associated uncertainties as a function of the noise structure. Finally we investigate the noise of AG values at frequencies higher than 1 cpd, where a white noise process usually dominates.

3.1. Noise Due to Setup of the AG

[13] The superconducting gravimeter C021 instrumental drift has been evaluated by fitting a first-order polynomial on the difference between the SG and the AG gravity data [*Francis et al.*, 2004a]. Removing the instrumental drift from the SG data, one obtains the actual gravity changes. The AG and drift-free SG time series are shown on Figure 1. The power spectra of the raw SG, the drift-free SG and the AG are on Figure 2. We can assume that the SG instrumental noise is dominated by environmental colored effects at periods longer than 1000 s and periods shorter than 100–50 s. This is similar to other SGs [*Banka and Crossley*, 1999]. Using the SG C021 time series as well as results published by *Banka and Crossley* [1999], we evaluated the SG instrumental white noise level to lie between 1 and 3 nm s^{-2} $Hz^{-1/2}$. As for the absolute gravimeter, the instrumental noise is usually between 50 and 80 nm s^{-2} drop-to-drop (DTD) noise at Membach. As the drops are repeated every 10 s, 100 to 200 times per hour, the 10 s sampling rate averaged on one hour is 36 to 18 s. In the best case this results in $\sqrt{2 \times (50^2 \text{ or } 80^2) \times 18} = 300$ or 480 nm s^{-2} $Hz^{-1/2}$ (see section 3.3), more than 2 orders of magnitude higher than a SG. Thus, after removing the SG drift, the SG is supposed to be perfect for all frequencies except the DC and so, the difference [AG-SG] can be considered to be the setup error in the AG.

[14] The histogram of the difference [AG-SG] is shown on Figure 3, where the absolute gravity measurements are compared to the average of the SG ones over the same time period. The distribution is normal with a standard deviation σ = 16 nm s^{-2}; this was confirmed by computing that 68.3% (respectively 95.5%, 99.7%) of AG values did not

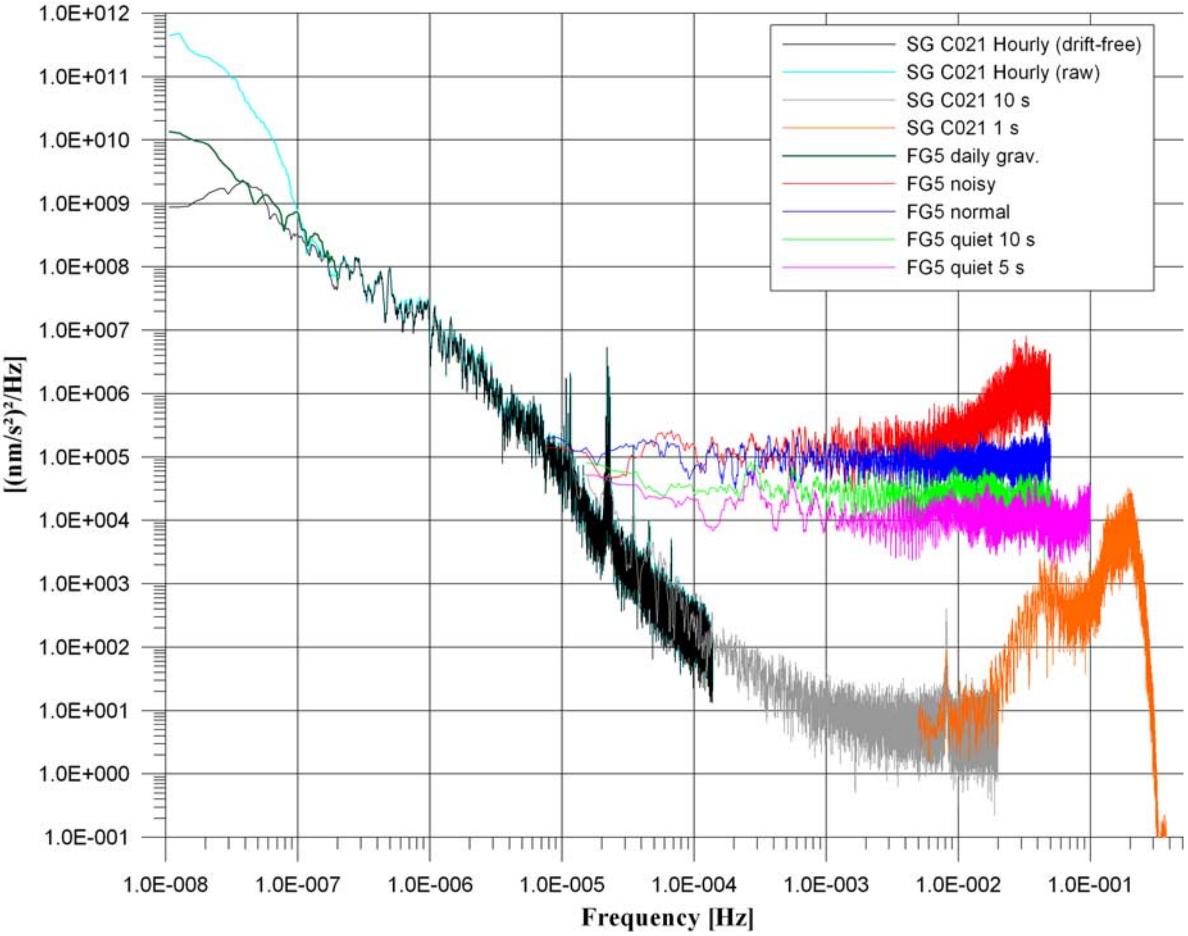

Figure 2. Membach: Power spectrum density (PSD) of the SG and AG data shown on Figure 1. The PSD AG time series is shown when the microseismic noise is very high (red), normal (blue) and quiet (purple is sampling rate of 1/5 s; green is sampling rate of 1/10 s). Dark green represents the spectrum from the 96 AG values. The SG spectrum is divided into four parts: black, using the hourly values from January 1996 to June 2004 after removing the SG instrumental drift by fitting a first-order polynomial on the difference between the SG and the AG gravity data; light blue, idem but without removing the drift; gray, using 10 s data during 31 days (January 2004); and orange, using 1 s data during 24 hours in April 1999. The environmental noise dominates SG observations for frequencies lower than 10^{-3} Hz and frequencies lower than 1–2.5 10^{-5} Hz (1–0.5 day period) for the AG measurements. The peaks due to the microseismic noise appear clearly, in spite of the low-pass filter [*Van Camp et al.*, 2000]. The peak at 8×10^{-3} Hz (122 s period) is due to a free oscillation of the levitating sphere, typical of the SGs. The peaks at 1.1, 2.3, 3.4, 4.6, and 5.8 × 10^{-5} Hz (1, 2, 3, 4, and 5 cpd) are due to imperfections in the correction of the tidal effects and harmonics of the solar heating tide S1.

differ from SG by more than 1σ (respectively, 2σ, 3σ). The distribution is slightly negatively skewed (−0.7 nm s^{-2}), which is not as surprising as some AG error sources systematically produce a decrease in gravity (poor vertical adjustment of the test beam, misalignment of the test and reference beams). However, it is still close to a normal distribution and confirms the quality of the FG5 202 such that it agrees with SG C021 at the 10–20 nm s^{-2} level. The AG setup instrumental noise process is therefore white at frequencies lower than 1 cycle per day.

[15] By coincidence, the setup noise is of the same order of magnitude as the root mean squared deviation of the sets, also called the set scatter or standard deviation of the sets. Another parameter commonly used is the experimental standard deviation of the mean $\sigma/\sqrt{N}$ (where σ is the set scatter and *N*, the number of sets), also called the "Measurement Precision". For the 96 Membach AG values (but also at other quiet stations), this parameter stabilizes between 0.5 and 2 nm s^{-2} after 12–24 hours of measurements and equals on average 1.5 ± 1.0 nm s^{-2}. Assuming a white noise process, this is equivalent to $\sqrt{2 \times 1.5^2 \times 86400}$ = 624(±330) nm s^{-2} $Hz^{-1/2}$ or in other words, to a drop-to-drop noise of 74 ± 39 nm s^{-2}. This is similar to the 50–80 nm s^{-2} mostly observed.

[16] We took the experimental standard deviation of the mean of SG data during the epochs corresponding to these 96 AG measurements and obtained on average 0.3 ± 0.2 nm s^{-2}. This represents the unmodeled real geophysical noise, and this is smaller than the AG measurement precision because the absolute gravimeter instrumental white noise dominates,

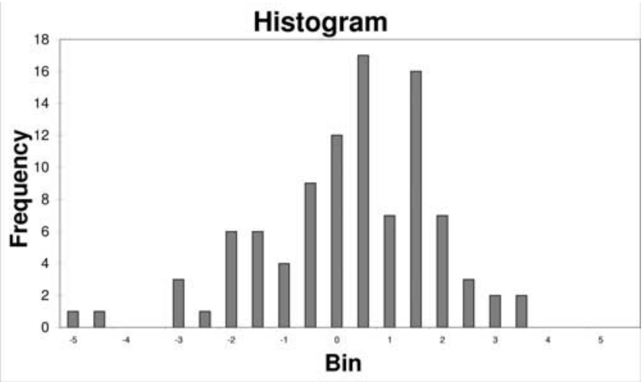

Figure 3. Histogram of [AG-SG], differences between the absolute gravity (AG) and superconducting gravity (SG) measurements at the Membach station shown on Figure 1. AG and SG measurements were averaged over the same time period. The distribution is negatively skewed (−0.7). The standard deviation $\sigma = 16$ nm s^{-2}; 68.3% (respectively 95.5%, 99.7%) of the AG data agree with the SG at the 1σ (respectively 2σ, 3σ level). Study performed on 96 AG values from February 1996 to June 2004.

even after 100 sets (4 days) and because the SG measures continuously.

[17] It is worth noting that each gravity value does not correspond systematically to a new setup. Sometimes the FG5 had been installed in the station for weeks, while two or more gravity values were measured. However, before measuring each gravity value, the instruments were still carefully controlled (checking the verticality, the overlap of the interfering beams etc.). At Membach 58% (and 82% at POL) of the gravity values were recorded without moving the FG5.

3.2. Noise at Low Frequencies (<1 cpd) and the Effect on the Linear Trend Estimate

[18] After removing the SG instrumental drift, AG and SG data produce the same spectra, as shown in Figure 2 and by *Crossley et al.* [2001]. However, as only 96 AG values are available, and as they are not evenly spaced, we used the drift corrected continuous SG data to investigate the geophysical power law process at low frequencies, which affects both instruments equally.

[19] With 10 days of combined AG and SG data the AG crossover frequency appears to be around 1 cpd. When the AG measurements are very quiet, the crossover frequency increases toward 6 cpd (7×10^{-5} Hz) ("quiet" on Figure 2). On the other hand, the SG crossover frequency at 10^{-3} Hz (1000 s period) is already visible with just 24 hours of data.

[20] Incidentally, we see that in the tidal band the noise level is $\sim$100−200 nm s^{-2} $Hz^{-1/2}$, as already pointed out by *Francis et al.* [1998]: they claimed 0.01−0.02 nm s^{-2} for integration periods of 2−3 years. Indeed, taking 2.5 years and 0.02 nm s^{-2}, this is equivalent to $\sqrt{2 \times 2.5 \times 365 \times 86400 \times 4\ 10^{-4}} = 170$ nm s^{-2} $Hz^{-1/2}$. They stated that this noise figure is similar to that obtained from an FG5 absolute gravimeter. We would like to be more explicit: the environmental noise dominates drift-free SG observations for frequencies lower than 10^{-3} Hz and frequencies lower than $1-2.5 \times 10^{-5}$ Hz (1−0.5 day period) for the AG measurements. In other words, the instrumental white noise still dominates AG data in the tidal band. This is not the case for the SG as the geophysical noise dominates its much lower instrumental noise. However, by coincidence, the AG instrumental noise is at the same order of magnitude as the environmental noise in this frequency band. When the absolute gravimeter samples at a higher rate, this does not improve the noise level for the frequencies lower than 1 cpd, as shown in Figure 2 (5 s data in Membach). At frequencies higher than 1 cpd, an improvement occurs, but in vain as the instrumental white noise still dominates. However, as shown in section 3.3, increasing the sampling rate can really be useful in particular stations like Ostend.

[21] To quantify the environmental noise we used the maximum likelihood estimation (MLE) technique to fit two types of stochastic model to the SG time series, power law noise and first-order Gauss Markov (FOGM) noise [*Gelb*, 1994]. We estimated the noise amplitudes of a white noise process plus power law model [*Williams*, 2003] while simultaneously solving for the spectral index, and we estimated the noise amplitudes of a white noise process plus FOGM noise model while simultaneously solving for the parameter β (equation (2)). Applying the power law noise model to the SG data shown in Figure 1 gives a spectral index, $\kappa = -2.4$. We also estimated κ by fitting a curve to the power spectrum (PSD) (Figure 2) of the SG data and found an index of −2.46. The spectrum appears to flatten toward low frequencies, so we restricted the fit to frequencies higher than 0.1 cpd and obtained an index of −2.50. Taking into account only frequencies smaller than 0.1 cpd we get an index of −1.25; using this spectral index we refer to fractional Brownian (FB) noise hereafter. At very low frequencies ($<10^{-7}$ Hz $\Longleftrightarrow$ 1 cycle per year (cpy)), the spectrum tends to white noise process ($\kappa = 0$). This is equivalent to first-order Gauss-Markov (FOGM) noise. This may be a consequence of removing a slope from the time series prior to the calculation of the PSD [*Johnson and Agnew*, 2000]. However, the fact that the PSD flattens at low frequencies may indicate that the noise processes affecting Membach are indeed stationary, or that it is a result of poor spectral resolution at very low frequency. In this case a power law process could appear again in the future when more data are available or the frequency where the PSD begins to flatten migrates to a lower value.

[22] The slope and its related uncertainty for the AG and SG Membach time series (Figure 1) calculated assuming various stochastic models is shown in Table 1. Taking into account different spectral indexes, using MLE, we calculated the slope and uncertainties using the AG and the drift-corrected SG Membach time series shown in Figure 1. If white noise process is assumed, the slope inferred from the AG is -4.3 ± 0.8 nm s^{-2} yr^{-1}, twice the result obtained from the SG but smaller than the -6.0 ± 1.0 nm s^{-2} yr^{-1} published by *Francis et al.* [2004a] based on the AG data. The time series was then 18 months shorter than the present one, but another problem is that the AG measurements are not uniformly spread out such that more weight was implicitly given to some epochs. In particular, numerous AG data were available in 1996 (Figure 1) when gravity was high: this explains why fitting the AG time series provides a higher trend. Thus it is better to weight AG data

Table 1. Uncertainties on the Slope Estimated Using the 96 (211) AG Values at the Membach (POL) Station for Different Power Law Processes[a]

Noise Model	Slope, nm s^{-2} yr^{-1}	Uncertainty, nm s^{-2} yr^{-1}
Membach SG Data		
White only (κ = 0)	−2.1	0.1
First-order Gauss Markov (equation (2))	−5.7	1.1
Flicker (κ = −1)	−5.3	0.7
Fractional Brownian (κ = −1.25)	−6.0	1.5
Random walk (κ = −2)	−7.6	21.9
Power law (κ = −2.4)	−9.7	57.7
Membach AG Data		
White only	−4.3	0.8
First-order Gauss Markov	−4.7	1.5
Fractional Brownian	−4.8	1.7
Flicker	−4.4	1.0
POL AG Data		
First-order Gauss Markov	10.3	1.4
Fractional Brownian	12.9	1.6
Flicker	13.2	0.9

[a] The uncertainties are calculated using the MLE method given by *Williams* [2003].

or to fit the SG time series as no epoch is preferred, but also to reject the white noise model assumed by *Francis et al.* [2004a] as this provided a too optimistic uncertainty. Our preferred noise model is either the FB or the FOGM for several reasons. First, the Random Walk model (and the power law noise model where the spectral index was estimated to be −2.4) results indicate that it would take on the order of 100 years to observe a gravity rate of change of 10 nm s^{-2} yr^{-1}. Given the excellent results from AG already shown [*Williams et al.*, 2001; *Larson and Van Dam*, 2000; *Lambert et al.*, 2001; *Francis et al.*, 2004a], we believe these uncertainties are too pessimistic. This argument was also put forward to reject some noise models for geodetic data by *Langbein* [2004]. It is quite easy to believe that geophysical processes should be stationary (and therefore have a flat spectrum at low frequencies) as there are probably some physical bounds to the range of these processes. Second, tests were performed by applying the MLE on simulated time series and looking at what frequency range the derived spectra agreed for the various models. It turns out that the derived spectra mostly agreed at frequencies close to the high end (perhaps an order or two magnitudes below the Nyquist frequency). So if the noise spectra did lower at low frequencies or the chosen spectral index was lower than predicted the MLE would tend to overestimate not only the spectral index but also the amplitude of that noise leading to widely pessimistic results. As a conclusion, the preferred models give an uncertainty of between 1 and 1.5 nm s^{-2} yr^{-1} and a slope of −6.0 nm s^{-2}.

[23] The spectral index probably reflects a mixture of geophysical origins such as hydrology, atmospheric effects and to a lesser extent, station instability. At the Membach station, this new statistical analysis confirms that there is a significant gravity rate of change of about −6.0 nm s^{-2} yr^{-1} if a FB or FOGM noise is considered.

3.3. Noise at High Frequencies (>1 cpd)

[24] At frequencies higher than 0.2–1 cycle per day, we can use the nearly continuous AG recordings corresponding to different occupation intervals of the instruments. Figure 2 shows four PSDs of AG time series recorded at the Membach station, when the microseismic noise was very high (DTD noise of 270 nm s^{-2}), normal (DTD noise of 80 nm s^{-2}) and low (DTD noise of 62 nm s^{-2}). In this case, the sampling rate was, exceptionally, one drop per 5 s but we also simulated the usual one drop per 10 s rate by taking every other point.

[25] The PSD of the microseismic activity usually shows two maxima at 0.06 and 0.17 Hz, as seen on the spectrum based on SG 1 s data. When this noise increases, an aliasing effect appears on Figure 2 when the AG is noisy, with a typical folding of the spectral content of the microseismic activity. When the drop to drop noise is smaller than around 50–80 nm s^{-2} (depending on the instrument quality) the instrumental white noise dominates. This was also checked by running the FG5 202 with another FG5 side by side: when the microseismic noise is high, the drop time series are clearly correlated. However, when the microseismic noise falls below 70 nm s^{-2}, the correlation between the series vanishes. This means that at least one instrument is then dominated by the white noise process.

[26] The PSD from recordings performed in Ostend (Belgian coastal city, sandy ground), POL (UK coastal town, on sandstone bedrock) and Jülich (German industrial area, soft sediments, mining activity, see *Van Camp* [2003a]) are shown in Figure 4 together with the Membach quiet case, already shown on Figure 2. The POL PSD is smoother than the others because it represents the average of over 200 separate PSDs, each calculated using a 24 hour long time series. Ostend provides a nice illustration of the folding of the microseismic noise: using the same time series, we took one data out of two, which simulates the 10 s sampling interval. This increased the PSD level by 20 dB and its shape became similar to the POL or Jülich ones. As already discussed, no such improvement was observed at the Membach station during quiet days, because the instrumental white noise process then dominated. Anyway, as seen in Figures 2 and 4, the aliasing effect is of very small amplitude at periods longer than 2 days. In noisy circumstances and if it is not possible to measure for more than 2–3 days, the aliasing effect can also be reduced by performing a drop every 5 s as in Ostend.

4. Estimating the Uncertainties on the Secular Trend

[27] In section 3.2, we estimated the uncertainties on the trend observed at Membach for different stochastic processes. At this station, we have on average one AG value per month. This is also the case at the POL reference station. However, as mentioned in section 1, several repeated AG measurement campaigns have been undertaken. The campaigns are usually repeated once a year, sometimes twice in the best case, and during the same season(s). Using the noise structure observed in Membach, we evaluated the expected uncertainties for one, two and four campaigns per year. With typical seasonal variations at Membach on the order of 40 nm s^{-2} and short-period, drop to drop, noise of 50–100 nm s^{-2}, this seams a reasonable choice. This is similar to other stations where AG, and if applicable, SG measurements are available [e.g., *Zerbini et al.*, 2001;

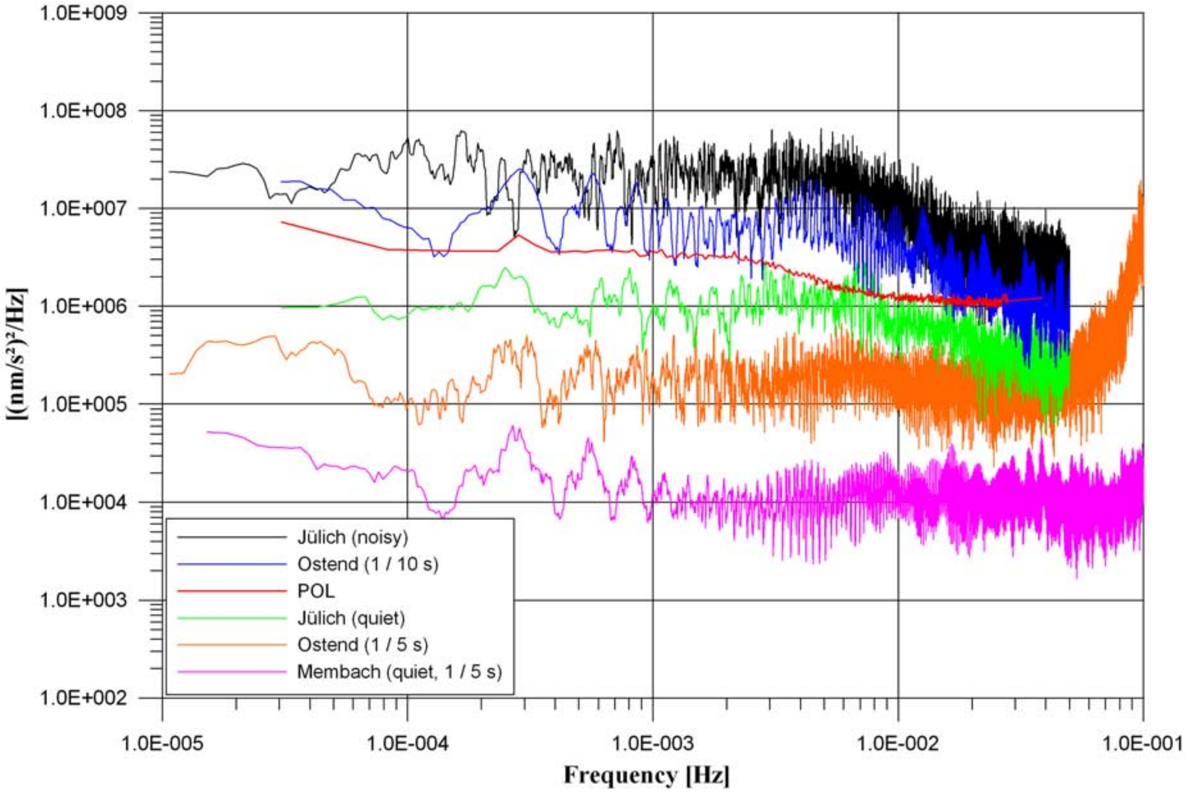

Figure 4. PSD of AG time series at Jülich station when the industrial noise is low (green) and very high (black). Red indicates average of 200 separate PSDs at POL, each calculated using a 24 hour long time series. Orange indicates PSD at Ostend (1 drop/5 s, 200 drops per set, 1 set per hour), and blue indicates the same by taking one data out of two. This evidences the aliasing of the microseismic noise. For comparison the PSD at Membach using the 1/5 s data recorded during quite days, already shown on Figure 2 in purple, is also shown.

Palinkas and Kostelecky, 2004]. The results are shown in Table 2. In the case of two measurements per year, taking into account the 16 nm s^{-2} setup noise, an uncertainty of 1 nm s^{-2} yr^{-1} (equivalent to 0.5 mm yr^{-1} considering the free air and Bouguer correction) is obtained after 14 years for the FOGM noise model and after 23 years for the FB model. If we ignore the setup noise, we save about 4 years in all cases and should reach 1 nm s^{-2} yr^{-1} after 8 years in the FOGM case and 20.4 years in the FB one. These values can be compared to continuous GPS measurements at midlatitude, where the precision in the vertical component is expected to reach 1 mm yr^{-1} (or 2 nm s^{-2}) after 6–8 years and 0.5 mm yr^{-1} (or 1 nm s^{-2}) after 13–16 years [*Williams et al.*, 2004]. This, however, does not include inaccuracies in the rates due to the definition of the reference frame which may be on the order of several millimeters per year. Performing one or four AG campaigns per year should not modify the uncertainties significantly, even if a couple of years can be saved in the case of the [flicker + setup] or [FOGM + setup] noise models.

5. Choosing the Error Bars

[28] Typically, when presenting AG (and other results) in figures the estimates are shown together with their formal errors. In section 3.1 we showed that the experimental standard deviation of the mean (or the "measurement precision"), usually close to 1 nm s^{-2}, provides a good estimate of the uncertainties due to environmental effects. We therefore suggest that the error bars should therefore be calculated by taking the square root of the sum of the squared experimental standard deviation of the mean plus the squared setup error (~16 nm s^{-2}).

[29] The uncertainties due to hydrology, error on the vertical gravity gradient, the tidal model, the laser, the clock and the barometer could also be added. Provided that they are regularly checked and calibrated the errors due to the clock, laser and barometer are around the 1 nm s^{-2} level. This is the same for atmospheric loading effects and tidal corrections, if an accurate model is used and if one makes at

Table 2. Time Necessary to Measure a Slope as a Function of the Number of AG Campaigns per Year[a]

Noise Model	Annual	Semiannual	Quarterly
Flicker	7.7	7.2	6.7
Flicker plus setup	15.1	12.7	10.8
FOGM	9.4	8.0	7.2
FOGM plus setup	16.5	13.7	11.5
FB	20.8	20.4	20.3
FB plus setup	24.7	23.4	22.4

[a] Time is in years. Slope has an uncertainty of 1 nm s^{-2} yr^{-1}. There are one to two or four campaigns per year. The setup noise is 16 nm s^{-2} (section 3.1). The Flicker, FOGM, and FB noise models were obtained using the Membach SG time series (section 3.2).

least 12 hours of measurements [*Van Camp*, 2003b]. The error due to the gradient depends on the use of the AG value. If repeated campaigns are performed using the same instrument, this should not matter as long as the vertical gravity gradient is stable in time. However, if *g* needs to be known for metrological purposes, the gradient error, the hydrological and long-period loading effects must be taken into account. Notice that the hydrological and loading errors can be reduced by measuring *g* whenever its value must be known. Of course this requires "a highly available" absolute gravimeter, or a SG.

[30] When more than one absolute gravimeter are used for repeated measurements campaigns, standard uncertainties of the mean gravity value (reference value) from intercomparison experiments could be used. They reached 28 nm s^{-2} in 1997 and 55 nm s^{-2} in 2001 [*Vitushkin et al.*, 2002], i.e., in the worst case, more than 3 times higher than the setup error. On the other hand, the recent 2003 intercomparison performed in Walferdange between 16 AGs provided a standard deviation similar to the setup error [*Francis et al.*, 2004b]. It is possible that with the improvement of the current techniques, the future standard uncertainties of the mean gravity value for intercomparison will reflect only the setup error. However, intercomparisons performed several times between the same AGs have evidenced some systematic differences [e.g., *Van Camp et al.*, 2003]. So, at this present time we prefer to remain careful. If the instruments show a difference at a common reference station before measuring later in the field at a common station, the best we could do is to take that difference into account. However, as intercomparisons occur at an instant in time (there is no connection between intercomparisons), an additional uncertainty of at least 16 nm s^{-2} should be added to the uncertainty budget when combining measurements from two different AGs.

6. Conclusions

[31] Using 8 years of SG and AG measurements at the Membach station and 10 years of AG measurements at POL, we estimated the noise affecting AG measurements in different frequency bands. We also provided an accurate estimate of the white noise due to setups of AG. Using these results, we estimated that repeated AG campaigns should allow one to constrain gravity rate of change with an uncertainty of 1 nm s^{-2} yr^{-1} (or 0.5 mm yr^{-1}) after 15–25 years, depending on the noise model. Therefore the absolute gravimeters are excellent tools for monitoring slow tectonic deformation. Like other geodetic measurements, particular care must be taken when estimating slopes and uncertainties for which a power law noise process must be preferred to a white noise process.

[32] In principle, since AG data are absolute and do not depend on any reference frame, the measurements should be usable for a long time. This is important for long-term projects like monitoring mean sea levels, or slow intraplate deformations. Combining AG and GPS provides information on the mass redistribution, which is of geophysical importance for constraining the viscosity of the upper mantle. Absolute gravimeters are also useful for certain types of instantaneous deformation such as coseismic movement [*Tanaka et al.*, 2001]. If it was possible to take one set per hour during a whole year, absolute gravimeters would provide the same quality as SGs at frequencies lower than 1 cpd. In practice, as these instruments are mobile and suffer from wear, the SG is better than the absolute gravimeter, even when seasonal variations have been clearly observed using sufficient AG measurements, for example at POL or Membach. However, as the absolute gravimeters travel and do not measure continuously, we must take into account (1) the white noise due to setup; (2) the possible differences between the absolute gravimeters if repeated measurements are not made with the same instrument; and (3) the aliasing due to the discontinuities between the AG observations. J. Nicolas (manuscript in preparation, 2005) clearly illustrated the difficulty in constraining seasonal variations by carrying out only 2 or 3 AG observations per year.

[33] SG time series being continuous are more suitable for studying seasonal effects and tidal phenomena [*Baker and Bos*, 2003]. At frequencies higher than 1 cpd, the AG instrumental noise dominates, therefore SGs are good instruments for measuring the gravest seismic free oscillations [*Widmer-Schnidrig*, 2003] or subseismic normal modes [*Crossley et al.*, 1999]. SG values are helpful in hydrology by providing information on groundwater mass, which is difficult to assess [*Crossley et al.*, 1999; *Meurers*, 2000; *Schmerge*, 2003]. A SG is also very useful to check the AG stability by determining if any observed offsets in the AG values are due to instrumental problems or are due to actual changes in gravity.

[34] On the other hand, regular AG measurements are absolutely necessary to remove the SG instrumental drift. Moreover, the AG is mobile and allows monitoring gravity and its variations on a regional scale. Finally, the absolute gravimeter, by nature, provides the gravity. This is very important for metrological purposes like the redefinition of mass [*Eichenberger et al.*, 2003]. Because of its absoluteness, an AG could theoretically go back to any undisturbed measured gravity point even after 100 years and make a measurement that is relevant. No relative geodetic techniques could compete with that.

[35] Absolute gravimeter, superconducting gravimeter, and GPS are very complementary geodetic techniques, and any geodetic reference station should include all of them.

[36] **Acknowledgments.** We are grateful to Trevor Baker, Thierry Camelbeeck, Marc Hendrickx, Tim Niebauer, Philippe Richard, Bernd Richter, and Richard Warburton for their fruitful comments and suggestions. M. Hendrickx, S. Castelein, and M. De Knijf have participated in the maintenance of the Membach station. We thank A. Ergen, J.-M. Delinte, and the staff of the Seismology Section (ROB) for the assistance in the numerous AG measurement campaigns. We would also like to thank Graham Jeffries and Danny McLaughlin for the collection of the UK POL data set. We are indebted to E. Kümmerle, M. Möllmann-Coers, and E. Pomplun for their warm welcome at the Jülich Forschungszentrum and J. Verstraeten at the Ostend station. The Belgian Physical Society is acknowledged for providing the funds which initiated this study. The work benefited from reviews by J. Merriam and an anonymous reviewer and from encouragements of T. van Dam.

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