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Accurate transfer function determination for superconducting gravimeters

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Abstract. The transfer function for the cryogenic gravimeter GWR-C021 operating at Membach (Belgium) has been experimentally determined by injecting known voltages into the control electronics of the system. The output of the gravimeter to the injected sine waves and step functions has been observed. This give a precise knowledge of the transfer function of the gravimeter. It allows one to reach a precision of better than 0.01 second in the phase response of the instrument, in agreement with the Global Geodynamics Project (GGP) requirements.

Introduction

In August 1995, the Royal Observatory of Belgium installed a superconducting gravimeter (SG) GWR-C021 at the station of Membach, in the East of Belgium. The signal to noise ratio (SNR) of SGs is very high over a wide frequency range, from seismic normal modes frequencies to the polar motion frequencies. However, the performances of those instruments are fully reached only if they are calibrated with an accuracy of 0.1% in amplitude and 0.01 s in phase. Such a calibration will provide constraints on oceanic tidal loading models [*GGP*, 1997a]. This is also necessary to evaluate the recent global Earth models which do not differ by more than 0.1% in their tidal gravimetric factors and only 0.01% in the phase [*Baker*, 1998]. Moreover, a precise phase determination of the tidal waves is useful for improving the estimate of the Nearly Diurnal Free Wobble (NDFW) quality factor Q [*Merriam*, 1995]. This provides a tool to estimate dissipative mechanisms inside the Earth [*Florsch and Hinderer*, 1998] [*Defraigne and Dehant*, 1994].

Presently 17 SG instruments participate in the Global Geodynamics Project (GGP), an international program of observations of the temporal variations of Earth's gravity field which will extend over a period of 6 years [*Crossley et al.*, 1999]. The GGP will contribute to improve mainly studies on seismic normal modes, Earth tides, ocean tidal loading, core modes, Slichter triplet and NDFW. The campaign began on July, 1, 1997.

Francis [1997] has already determined the amplitude calibration factor of the SG-C021. This was done by comparing SG data with simultaneous registrations of the FG5-202 absolute gravity meter. The amplitude calibration is known with a precision of 0.1% in the tidal band. Unfortunately, this experiment does not allow determination of the instrumental phase lag due to the noise of the absolute gravity data. In the present paper, we describe the method we used to determine the transfer function (response in amplitude and phase) of the SG C021. We focus on the phase lag (or time lag, if expressed in seconds). We also prove that the amplitude response varies for period shorter than 1000 s i.e. the calibration factor is frequency dependent at short periods.

The C021 superconducting gravimeter

A superconducting gravimeter consists of a hollow superconducting sphere that levitates in a persistent magnetic field. The frictionless bearing of the mass and the stability of the magnetic field generated by superconducting coils provide a highly sensitive gravimeter which is stable for long periods. An electrostatic capacitive device detects the vertical position changes of the levitating sphere and a magnetic feedback force maintains the sphere at a fixed position. SGs are equipped with an electronics card ("gravity control card") that contains the feedback integrator whose voltage is proportional to acceleration changes. This voltage is available from different low-pass and band-pass filters. In Membach, we used the two following low-pass filters: the 6 pole Tide filter (corner period at 72 s) and the 2 pole Gravity Signal (GS) filter (corner period at 1 s) [*Van Camp*, 1998]. In December 1997, the gravity control card was replaced by a card with a new 8 pole low-pass filter to fulfil the GGP recommendations [*GGP*, 1997b]. This filter has a cutoff period of 16 s and is called GGP1.

Theory

There are essentialy two experimental methods to determine the transfer function of SGs. The first one consists in applying external forces by placing the instrument on an oscillating platform [*Richter*, 1995] but nothing about the instrumental phase has been published. However, Richter (pers. comm., 1999) plans to determine the phase delay but does not expect a precision better than 0.1 s. The second

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Table 1. Time lags obtained by step response and sine waves for the Tide, GS, GGP1 filters and the feedback integrator. For sine waves, we give the number of files used to calculate the time lags that are averaged at 500 (excepted Tide), 1000 and 2000 s and for steps, the length of the data sets analysed. LSQ means application of a LSQ low-pass filter. For the step response, we give the average of the time lags at the period of 2000 s. All time lags take into account the delay induced by the permanent data acquisition systems.

Experiment	Sine-Waves:	Time lag	Steps:	Time lag
	used files	[s]	length	[s]
Tide (96)	5	38.580±0.041	4 min	38.563±0.050
GS (96)	6	3.382±0.039	4 min	3.324±0.075
			2 min	3.361±0.033
			2 min + LSQ	3.363±0.011
Integrator (96)	6	0.524±0.033	1 min	0.517±0.047
GGP1 (98)	9	12.103±0.002	2 min 50 sec + LSQ	12.101±0.003

method consist in injecting known voltages into the gravimeter feedback loop. Comparison between the input and output signals gives the transfer function. In this paper, the injected voltages are step functions and sine waves, and both should give the same result. Because reaching a precision of 0.01 s for the time lag is difficult, especially at long period (T > 500 s), it is worthy to compare both methods in order check the quality of the results. It allows also one to determine the transfer function of the permanent data acquisition systems. Finally, it gives an opportunity to study how the noise affects the different methods.

Step response method

In the step response method, one applies a step voltage into the gravimeter as an input function, and one observes the response of the system to the input step as output. The Fourier-spectrum of the differentiated step response function gives the frequency transfer function of the system. Unlike sine waves, this method has the advantage that the derivation of the transfer function from the differentiated step response does not use any parametric model for the instrument [*Richter and Wenzel*, 1991]. Before any analysis of the step responses,Earth tides effects are removed from SG data by subtracting a synthetic tide determined by previous registrations [*Francis*, 1997].

Sine-wave method

A second method for determining the instrument response consists of applying a sine wave with known period T as the input function. The data have been analysed by

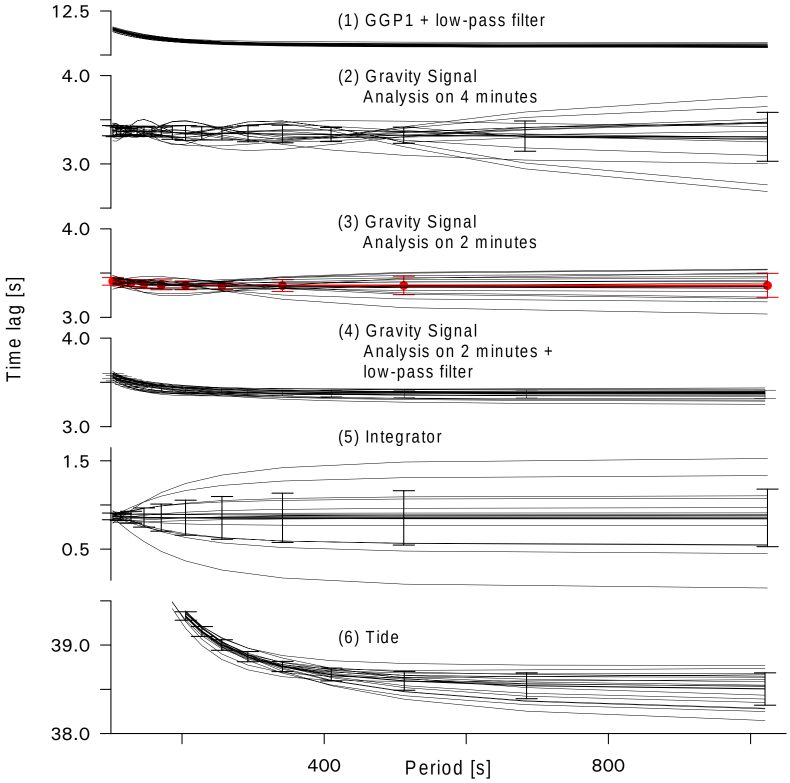

Figure 1. Determination of the time lag by the step response method. (1) GGP1, analysis on 2 min 50 s after LSQ filtering (2) GS, analysis on 4 minutes - (3) 2 minutes - (4) 2 minutes after LSQ filtering (5) Integrator, analysis on 1 minute (6) Tide, analysis on 4 minutes.

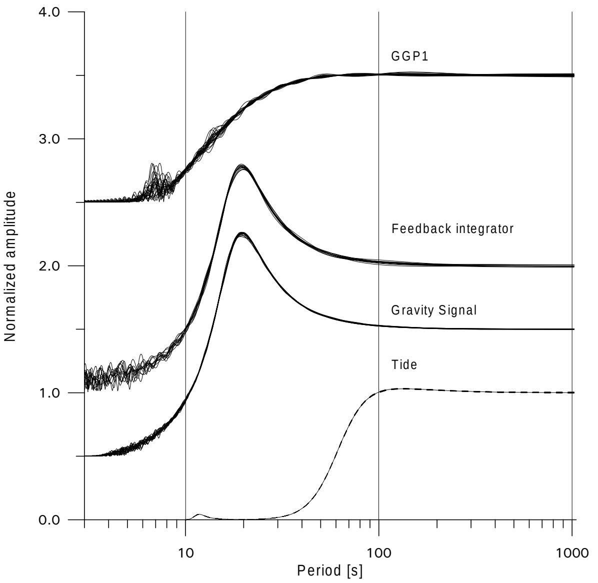

Figure 2. Normalised amplitude response of GGP1 (analysis on 1 min 30 s), GS (analysis on 2 min), Integrator (analysis on 1 min), Tide (analysis on 4 minutes). For a good legibility, an arbitrary shift of 0.5, 1.0 and 2.5 is applied to GS, Integrator and GGP1, respectively.

fitting both the input and output signals on the function $a(T)\sin(2\pi t/T)+b(T)\cos(2\pi t/T)+P_6(T)$. In this equation P_6 is a 6th degree polynomial necessary to remove Earth tides and drifts. We found that removing Earth tides by subtraction of a synthetic tide gives similar results. The amplitude ratios and the phase differences of the fitted input and output sinusoidal waves provide the instrumental transfer function.

In order to evaluate the noise effect on sine-wave measurements at periods longer than 100 s, we use a bootstrap method.

Experiments

The gravity card change modifies the instrumental characteristics. Hence, two transfer function experiments were made: one in July, 1996 and one in February, 1998. The added voltage creates electromagnetic forces on the gravimeter sphere, proportional to the voltages applied to feedback coil. The reaction of the gravimeter to the electromagnetic forces has been measured by digitizing the input voltages and the output voltages of the Tide, Gravity Signal and GGP1 low-pass filters as well as the feedback integrator.

For the July 1996 experiment, we used a Wavetek generator model 133 to produce the sine waves and a TTL switch controlled by an external DCF77 receiver clock to generate the step functions.

For the February 1998 experiment, we used a 12 bits D/A converter (Datel DAC HZ12 BGC) controlled by a PC and a DCF77 clock to generate step functions and sine waves. This system has an internal accuracy of 0.01 s on 2000 s sine waves.

To search the transfer function in the tidal band, we should inject sine waves whose periods equal 12 h or 24 hours. This is practically difficult to realise because: (1) about ten oscillations are necessary for a good signal-to-noise ratio and such an experiment would last several days during which earthquakes, ocean and atmospheric effects would cause perturbations, (2) the Wavetek generator is limited to periods shorter than 1 hour, (3) longer periods would corrupt data over too many days.

On the other hand, simulation analysis of the response of the GWR analog low-pass filters indicates that the Tide filter is flat from 1000 s and the GGP1 one, from 500 s [*Van Camp*, 1998]. Our experiment proves also that the integrator response is also flat from 500 s.

In the sine-wave method, the transfer function is extrapolated in the tidal band by averaging the results obtained at 1000 s and 2000 s for the Tide output and at 500 s, 1000 s and 2000 s for the GS and GGP1 outputs. The small magnitude of the standard deviations on these averages is a good indication of the flatness of the transfer function at long period (Table 1) . This problem also concerns the step response analysis, as ideally one should analyse a 12 h or 24 h long step in order to know the gravimeter response at such periods. We have limited the analysis to 2000 s, considering that the transfer function is flat from this period to the tidal band.

Results

Step function

In July 1996, 18 steps functions were injected into the gravimeter with a voltage varying from 0 to 4 Volts: 9 upward and 9 downward, each of them lasted 8 minutes and no disparities were observed between steps going up or down. For the Tide output, each step was analysed on the 4 following minutes. Time lags are presented in Figure 1. For GS output, the same analysis gives more perturbed results because the high frequency noise, especially microseismic noise, is less attenuated by the GS filter than by the Tide filter. As the GS filter delay is shorter than the tide filter delay, we re-tried the calculations on 2 minutes. This reduces the noise effect such that the dispersion is shorter than for the tide output. A further improvement was ob-

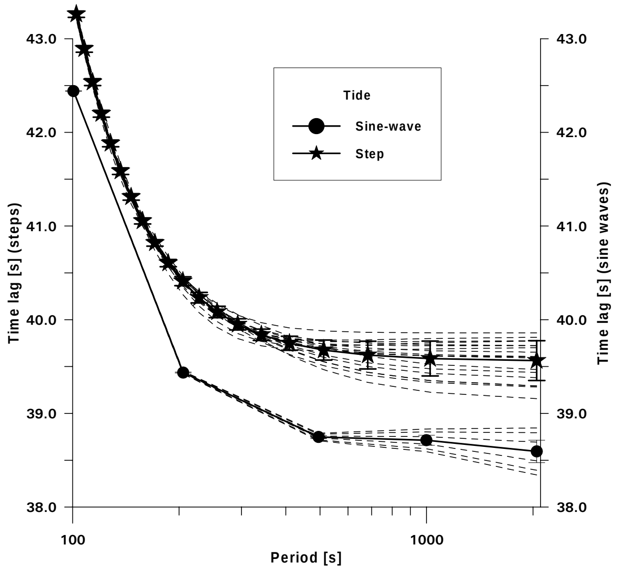

Figure 3. Tide: time lag calculated by the step response method (up) and by sine waves (down) where the noise effect is calculated by bootstrapping. For a good legibility, delays obtained from steps are artificially shifted by 0.5 s

tained by applying a LSQ filter (corner period 45 s, length = 61 s) [*Bloomfield*, 1976].

To assess the noise effect we can look at the standard deviations (Table 1) of the time lags. The time lags from GS, GGP1 and Tide outputs include the delay of their data acquisition systems, which are 2.635 s for GS and GGP1, and 0.5 s for Tide. We have also determined the time lag of the integrator, which is flat beyond 100 s.

In February 1998, 25 step functions were injected using different amplitudes, from 4V to 9V and starting at different levels, from -8 to +4 V. We did not notice effects due to the size and the starting point of the steps. We performed the analysis on 3 minutes.

In Figure 2, the amplitude responses of the GGP1, GS, integrator and Tide outputs show that the calibration factors are frequency dependent. In addition, the gravimeter shows a resonance at 19.7s, a period nearly rejected by the Tide and the GGP1 filters. The integrator output is noisier than the GS output augmented with an analog 1 s low-pass filter followed by a numerical LSQ filter (corner period at 5 s). Due to a higher microseismic noise in February 1998, the GGP1 output response is the noisiest despite its lower cut-off period.

Sine-waves

During the July 1996 experiment sine waves with amplitudes of 3.54 V peak-to-peak and periods of 50, 100, 200, 500, 1000 and 2000 s were injected into the SG.

In February 1998, we used more periods between 5 and 2000 s. For some periods, we worked with different amplitudes (4 to 7 V peak-to-peak) but we did not notice any significant differences between the results.

Time lags and amplitude responses are very similar to these obtained in the step response experiment. Figure 3 shows the time lag dispersion for Tide output obtained by bootstrapping where the different curves give the limits containing 50%, 95% and 98% of the results. Just like the step response method at the longer the periods, the higher the noise level. We see on Figure 3 that steps are more affected by the dispersion than the sine-waves. Note that bootstrapping give a similar dispersion for GS and the integrator in spite of the fact that these outputs are less filtered.

It is difficult to perform several sine waves experiments at long period, but even at 2000 s and 1000 s the experiments were long enough to subdivide files. We obtained so a minimum of 3 files for each periods (6 at 100 s) that enable the calculation of an average and a standard deviation that is very similar to the one given by bootstrapping.

For both the step response and the sine waves method, the accuracy of the transfer function determination has been improved by one order of magnitude during the 1998 experiment, compared with the work done in 1996. This improvement results from hardware upgrades.

Conclusion

We have measured the transfer function of the SG by injecting artificial signals to the feedback loop. The SG has been found to be frequency dependent. However, the amplitude and the time lag response of the Tide output is flat from 1000 s and for the Gravity Signal and the GGP1 ones, from 500 s. Our experiments have also shown that it is not only important to determine the transfer function of the instrument itself, but also to check the data acquisition systems that could also produce unexpected time lags. Both sine waves and step functions are suitable methods for determining the GWR superconducting gravimeter time lag with accuracy better than 0.01 s. The discrepancy between the two methods lies within the error bars. The step response method is more sensitive to the noise. The sine wave method is less noise-sensitive but longer to complete experiment.

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References

Baker, T.F., Tidal gravity observations and Earth tide models, in *Proc. 13th Int. Symp. Earth Tides*, edited by B. Ducarme, pp. 287-294, Bruxelles, 1998.

Bloomfield, P. *Fourier analysis of time series: an introduction*, John Wiley & Sons, New-York, 1976.

Crossley, D., J. Hinderer, G. Casula, O. Francis, H.-T. Hsu, Y. Imanishi, G. Jentzch, J. Kääriänen, B. Meurers, J. Neumeyer, S. Pagiatakis, B. Richter, K. Shibuya, T. Sato and T. van Dam, The Global Geodynamics Project, *EOS Trans. AGU, 80(11)*, 121, 125-126, 1999.

Defraigne, P. and V. Dehant, Stacking gravity tide measurements and nutation observations in order to determine the complex eigenfrequency of the nearly diurnal free wobble, *J. Geophys. Res., 99*, 9203-9213, 1994.

Florsch, N. and J. Hinderer, Estimation of the free core nutation Q factor from tidal analysis, in *Proc. 13th Int. Symp. Earth Tides*, edited by B. Ducarme, pp. 315-322, Bruxelles, 1998.

Francis, O., Calibration of the C021 superconducting gravimeter in Membach (Belgium) using 47 days of absolute gravity measurements. *Int. Ass. of Geodesy Symp., 117*, 212-218, Springer-Verlag, 1997.

GGP, *GGP Newsletter #4*, edited by D. Crossley and J. Hinderer, June 20, 1997, 1997a.

GGP, *GGP Newsletter #5*, edited by D. Crossley and J. Hinderer, Sept. 10, 1997, 1997b.

Merriam, J., The atmospheric pressure correction in gravity at Cantley, Quebec, in *Proc. 12th Int. Symp. Earth Tides, Beijing 1993*, edited. H.T. Hsu, pp. 161-168, Beijing, 1995.

Richter, B., H. Wilmes and I. Nowak, The Frankfurt calibration system for relative gravimeters, *Metrologia, 32*, 217-223, 1995.

Richter, B. and H.-G. Wenzel, Precise instrumental phase lag determination by the step response method, *Bull. Inf. Marées Terrestres, 111*, 8032-8052, 1991.

Van Camp, M., Qualification d'un gravimètre cryogénique pour les périodes supérieures à cent secondes, PhD thesis, 208 pp., Royal Observatory of Belgium, December 1998.

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