Effects of earth’s spherical curvature and radial heterogeneity in dislocation studies—for a point dislocation

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Received 2 December 2001; revised 11 March 2002; accepted 4 April 2002; published 29 June 2002.

[1] To the present, dislocation theories for a homogeneous half-space are often used to calculate or interpret displacements and gravity changes caused by an earthquake or to inverse a seismic fault model. However, far-field effects of spherical curvature and radial heterogeneity have to be considered. In this research, Okada [1985] and Sun et al. [1996] dislocation theories are used to calculate displacements caused by four independent dislocations in three earth models: a homogeneous half-space, a homogeneous sphere, and a heterogeneous sphere. Effects of spherical curvature and radial heterogeneity are investigated through comparison of displacements. Results show that effects of both sphericity and stratification are very large. The stratified effect reaches a discrepancy of more than 25% everywhere on the surface of the earth, including the near field. INDEX TERMS: 1242 Geodesy and Gravity: Seismic deformations (7205); 1299 Geodesy and Gravity: General or miscellaneous; 1212 Geodesy and Gravity: Earth’s interior—dynamics (8115, 8120); 1645 Global Change: Solid Earth; 7299 Seismology: General or miscellaneous

1. Introduction

[2] The most remarkable point of progress in seismology was the discovery that earthquakes result from fault movement (dislocation). Accordingly, dislocation theory plays an important role in linking earthquakes or volcanic events and geophysical phenomena observed by geodetic methods; i.e., dislocation theory can be used to interpret geodetic deformations, such as displacements, strains, and gravity changes. Conversely, dislocation theory can be used to infer a seismic fault with geodetic information as boundary conditions. So far, several dislocation theories have been proposed. Some of them are valid for a homogeneous half-space, such as those of Okada [1985] and Okubo [1991, 1992]; some of them are good for a homogeneous sphere or a heterogeneous sphere, like those of Sun and Okubo [1993, 1998], Sabadini et al. [1995], Piersanti et al. [1995, 1997] and Sun et al. [1996]. Some of them can be used for an elastic medium [Okada, 1985; Okubo, 1991, 1992; Sun and Okubo, 1993, 1998; Sun et al., 1996], and some of them can be applied for a viscoelastic body [Sabadini et al., 1995; Piersanti et al., 1995, 1997].

[3] For most cases, since we are interested in co-seismic deformation, a dislocation theory for a perfect elastic body is efficient. Due to mathematical simplicity of theories for a homogeneous half-space, Okada’s [1985] theory for calculating displacement, tilt, and strain, and Okubo’s [1991, 1992] theory for evaluating potential and gravity changes have been widely used as standard formulations. They are often used to study geodetic deformations caused by an earthquake or a volcano. For example, Okubo et al. [1991] successfully applied the dislocation theory of Okubo [1991, 1992] to study gravity changes caused by the 1989 earthquake swarm and submarine eruption off Ito, Japan.

[4] However, since the real earth more closely approximates a homogeneous sphere than a homogeneous half-space, spherical curvature and radial stratification bring serious errors when half-space theories are used to analyze, at least, far-field deformation. Therefore, a theoretical investigation is desirable to clarify effects of sphericity and stratified structure of the earth. Amelung and Wolf [1994] studied the spherical effect problem for surface loading. They compared spherical-earth models with incremental gravitational force (IGF) and plate-earth models without IGF and found that errors due to neglect of sphericity and the IGF partially compensate earth other.

[5] Sabadini and Vermeersen [1997] investigated influence of lithospheric and mantle stratification on global (co-) post-seismic deformation based on the normal mode technique. They found that the mantle viscosity structure has a major influence on post-seismic deformation in a far field. However, their discussion was limited to a dip-slip source. Also, deformation at and near an epicenter (say, 100 km), the area with greatest deformation for a tensile source, is difficult to clarify with the normal mode technique.

[6] This study investigates effects of sphericity and stratification by applying different dislocation theories. For simplifying discussions, but without losing generality, we limit our concentration to displacements caused by a point dislocation. Discussions for other geophysical deformations like gravity and strain would be similar. We also constrain our discussion to co-seismic deformation so that corresponding elastic dislocation theories of Okada [1985] and Sun et al. [1996] can be used. This study shows that effects of both sphericity and stratification are remarkably large; even for a near field, the stratified effect is also quite obvious, reaching a discrepancy of 25%. Therefore, a dislocation theory for a heterogeneous sphere, e.g., SNREI (Spherically Symmetric, Non-rotating, Elastic and Isotropic) model [Dahlen, 1968], is recommended.

2. Dislocation Theories for Different Earth Models

[7] A dislocation model defined in Figure 1 is applicable to all earth models in this study. It is expressed by the slip vector \( \mathbf{v} \), normal \( \mathbf{n} \), slip angle \( \lambda \), and dip angle \( \delta \) in the coordinate system \((x, y, r)\). For a spherical earth model, the coordinate system \((x, y, r)\) is replaced by \((\varphi = 0^\circ, \varphi = 90^\circ, r)\). The source is defined along coordinate \(r\). Relative movement of the two fault sides is defined as \((U/2) - (-U/2) = U\). Differential area of the fault is denoted by \(dS\). Note that for a tensile opening, the slip vector and the normal become the same, i.e., \(v = n\). To study Earth responses to seismic dislocation, the earth is often simplified into some simple geometric model: a homogeneous half-space, a homogeneous sphere, or a heterogeneous sphere, as demonstrated in Figure 2.

[8] Homogeneous half-space is the simplest earth model, which assumes the earth to be a half space filled with a homogeneous elastic medium. For such a simple earth model, it is easy to give mathematical expressions for seismic responses of the medium. For example, Okada [1985] presented a complete set of analytical formulae for calculating displacements, tilts, and strains caused by shear and tensile dislocations. Okubo [1991, 1992] proposed expressions in closed form to describe potential and gravity changes due to dislocations. Due to their mathematical simplicity,
these dislocation theories [Okada, 1985; Okubo, 1991, 1992] have been widely applied up to the present day to study or inverse seismic faults. However, validity of these theories is limited to a near field because the Earth's curvature and radial heterogeneity are not considered at all. Since modern geodesy is able to detect far field deformations, even a global co-seismic deformation, a dislocation theory for a more realistic earth model is needed to interpret far field deformation.

[9] The homogeneous sphere is obviously superior to homogeneous half-space since it includes curvature of the earth. Since it does not contain a stratified structure of the earth, we can observe the effect of sphericity by comparing results calculated for a homogeneous half-space and a homogeneous sphere. For this purpose, media parameters for the two models can be taken as identical.

[10] Comparing the homogeneous half-space and the homogeneous sphere models, the heterogeneous sphere, such as the 1066A [Gilbert and Dziewonski, 1975] or the PREM [Dziewonski and Anderson, 1981], is the most realistic model, since it contains both sphericity and stratification of the earth. For such an earth model, Sun and Okubo [1993, 1998] and Sun et al. [1996] presented dislocation theories to calculate co-seismic displacements and gravity changes. These theories are valid for the entire earth surface since they include earth sphericity and stratification.

[11] In this research, we take parameters of the top layer of the 1066A earth model as those of the homogeneous half-space and the homogeneous sphere. This is not the only choice possible; for example, if means of parameters have been used, better results may be obtained for certain lateral distances from the source. However, the choice in this research is to have a unique comparison between the three models, with a known check standard at the epicenter, since a sphere deformation limit is theoretically equal to that of a half-space when the model media parameters are identical. At the same time, by this choice, sphericity and stratification effects of the earth can be clearly identified.

3. Comparison Between a Homogeneous Half-Space and a SNREI Model

[12] At first, as a general case, we consider a homogeneous half-space and a SNREI earth model. The 1066A earth model is used for the dislocation theory of Sun et al. [1996]; parameters of the top layer of the 1066A are taken as the medium of the homogeneous half-space used for the dislocation theory of Okada [1985]. For a vertical strike-slip source at a depth of 32 km, vertical displacements at the surface of the earth are calculated for both models. Comparative results are plotted in Figure 3. Solid lines represent displacements calculated for the 1066A model, dotted lines show homogeneous half-space results. Figure 3a plots results for a near field within 2°, and Figure 3b shows results for a far field between 50° to 180°.

[13] Figure 3 shows that distribution patterns of the two models are almost the same, but have one rather large discrepancy. Almost the entire near field shows a difference of more than 20%. Between epicentral distances 0.9° and 1.4°, the two results are opposite in sign. This implies that a totally different (or wrong) result is obtained if the homogeneous half-space theory is used in this epicentral period. In the far field, from 60° to 180°, the two results are different in sign due to geometrical difference of the two earth models. In this case, the homogeneous half-space theory becomes absolutely invalid. Therefore, we learn from Figure 3 that for an epicentral distance beyond 1°, it is fallacious to use dislocation theory for a homogeneous half-space. To obtain a result with accuracy higher than 20%, dislocation theory for a SNREI model is necessary.

[14] Discrepancy obtained in comparison of Figure 3 includes effects caused by both spherical curvature (plus self-gravitation) and stratified structure. In the following sections, we discuss the two effects, separately.

4. Effects of the Earth’s Spherical Curvature

[15] To study effects of sphericity, two earth models are used: a homogeneous half-space and a homogeneous sphere. Numerical calculations are made for four independent seismic sources: two shear strikes and two tensile openings, i.e., a vertical strike-slip fault, a vertical dip-slip fault, a horizontal tensile opening (δ = 90°) and a vertical tensile opening (δ = 0°). Media parameters used in both models are equal to those of the top layer of the 1066A earth model. Fault size (L = Length, W = Width, U = Dislocation) is taken as L × W × U = 10° m³. Source depth is considered for different values; so it is possible to observe effect variance with changed depth.

[16] Figure 4a gives vertical displacements caused by the four source types at a depth of 20 km buried in the half space and the
homogeneous sphere, respectively. From top to bottom are results for vertical strike-slip, vertical dip-slip, horizontal tensile opening, and vertical tensile opening, respectively. The x-axis shows the epicentral distance up to 200 km. The y-axis shows magnitude of displacement expressed in centimeters. The solid line indicates results of the homogeneous sphere, while the dotted line shows results of the homogeneous half-space. Since results of the two earth models are almost the same, it is difficult to identify their differences since they overlap one another. This indicates that the effect of sphericity is very small for a shallow seismic source. However, as depth increases, their discrepancy increases. This can be seen in Figure 4b, showing the same comparison but for a 300 km depth. Figure 4 shows that discrepancies exist in all types of seismic sources, but they are extremely large for tensile openings, especially the tensile opening at a vertical fault ($\eta = 90^\circ$).

Figure 4. Comparison of vertical displacements calculated for four types of dislocations at depth of (a) 20 km and (b) 300 km buried in the homogeneous half-space and the homogeneous sphere. From top to bottom are results for vertical strike-slip, vertical dip-slip, horizontal tensile opening and vertical tensile opening, respectively. The solid line represents homogeneous sphere results; the dotted line shows homogeneous half-space results.

5. Effects of Earth's Radial Heterogeneity

To study the effect of stratified structure, we consider the homogeneous sphere and the heterogeneous sphere (1066A model), and compare results calculated from the two models. Numerical calculations are made using the same parameters as the above section. Figure 5a shows vertical displacements calculated for four types of seismic sources at a 20 km depth. Similarly, the solid line shows 1066A earth model results; the dotted line is for homogeneous sphere results. It is plain from the figure that discrepancies between the two models exceed 25% almost everywhere, including the epicenter. Discrepancies caused by stratified structure are much larger than those of sphericity. Hence, it may be concluded that the half-space dislocation theory may create error of 25%.

When source depth increases, discrepancies grow larger, as shown in Figure 5b, which plots vertical displacements for a source at a 300 km depth. This behavior mimics sphericity effects. Tensile opening discrepancy is extremely large, even showing an opposite sign at the epicenter.

In the case of shear dislocation, effects at the epicenter are zero. This holds true for any earth model because a strike-slip fault causes a four-quadrant distribution pattern, and its center ($\theta = 0^\circ$)

Figure 5. Comparison of vertical displacements calculated for four types of dislocations at (a) 20 km and (b) 300 km depth buried in the homogeneous sphere and the 1066A model.
must be zero; the vertical dip-slip fault causes a two-quadrant distribution, and the center locates at the node line. However, in the case of tensile opening, the curvature effect occurs for deep sources, but not for shallow ones, while the stratified effect occurs for any source depth.

6. A Case Study

[23] In this section, a case study on far field deformations caused by the 2000 Miyakejima volcanic and seismic activities further proves importance of dislocation theory for a heterogeneous sphere. The Miyakejima (an island among the Izu islands of Japan) volcano started its activities on June 26, 2000. An earthquake swarm followed in several days in the region 20 to 50 km away from the volcano. Volcanic and earthquake activities lasted for two months, accompanying a large tensile opening (dyke intrusion) and shear dislocations (four large earthquakes with magnitudes greater than 6.0). Many geophysical changes and crustal deformations were observed in and around Miyakejima, including gravity and electromagnetic variations [Furuya et al., 2001; Kaidzu et al., 2000; Kikuchi et al., 2001; Nakada et al., 2001; Sasai et al., 2001]. Aside from significant near field displacements observed by a GPS network, far field displacements were also successfully detected by a regional VLBI network around Tokyo, i.e., the KSP network of the Communication Research Laboratory, located at about 100–300 km distance from the source. Details concerning the VLBI network and far field deformation are found in Okubo et al. [2001]. Observed vertical displacement and magnitude of horizontal displacement at Tateyama were –1 cm and 4.9 cm, respectively; the baseline between Tateyama and Kashima changed –4.5 cm (Table 1).

[24] To interpret far field deformation, Okada’s [1985] and Sun et al.’s [1996] dislocation theories are applied to calculate displacements and baseline change. To observe effects of sphericity and stratification, the 1066A earth model (Model A) and the revised top layer replaced with lower Poisson’s ratio (Model S) are used for Sun et al. [1996] theory. Corresponding medium parameters of the top layer are taken for half space models (Models A’ and S’) to be used for Okada’s [1985] theory. Numerical results calculated for the four models are listed in Table 1. It is interesting to note from Table 1 that dislocation theory for a homogeneous half-space is not sensitive to the earth model; i.e., it cannot reflect a change in medium. However, Sun et al.’s [1996] theory gives obviously different results for the two earth models. This difference is considered to reflect effects of the earth models. On the other hand, vertical displacement calculated by Model S’ agrees well with the observed one. Again, the above results and discussions indicate that the dislocation theory for a heterogeneous sphere is better than that for a homogeneous half-space.

Table 1. Observed and Calculated Displacements at Tateyama and Baseline Change Between Tateyama and Kashima Caused by 2000 Miyakejima Volcanic and Seismic Activities

<table>
<thead>
<tr>
<th></th>
<th>Vertical</th>
<th>Horizontal</th>
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</thead>
<tbody>
<tr>
<td>Observed</td>
<td>–1.0</td>
<td>4.9</td>
</tr>
<tr>
<td>Half-Space Model A’</td>
<td>–0.3</td>
<td>5.6</td>
</tr>
<tr>
<td>Model S’</td>
<td>–0.6</td>
<td>5.8</td>
</tr>
<tr>
<td>Spherical</td>
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<td>1.3</td>
</tr>
<tr>
<td>Model A</td>
<td>–1.0</td>
<td>5.6</td>
</tr>
<tr>
<td>Model S</td>
<td>–1.0</td>
<td>4.9</td>
</tr>
</tbody>
</table>

*Unit: cm.

[25] Acknowledgments. This research was supported financially by a JSPS research grant (C13640420). Constructive comments on an earlier draft of the paper by two anonymous referees are gratefully acknowledged.

References