Metrologia 42 (2005) S138–S144 doi:10.1088/0026-1394/42/3/S14

Relativistic theory for time comparisons: a review

Gérard Petit[1] **and Peter Wolf**[1,2]

[1] BIPM, Pavillon de Breteuil, F-92312 Sèvres Cedex, France
[2] SYRTE, Observatoire de Paris, 61 av. de l'Observatoire, F-75014 Paris, France

E-mail: gpetit@bipm.org

Received 7 March 2005
Published 7 June 2005
Online at stacks.iop.org/Met/42/S138

Abstract
Soon after the advent of atomic clocks 50 years ago, their unprecedented uncertainties and their widespread use required that the definition of time scales and clock comparison procedures be considered rigorously within the theoretical framework of general relativity. We review the present procedures and show that the current relativistic modelling is adequate for the need of clocks and time transfer techniques. Accounting for the expected improvements in clock and time transfer technology, we investigate some possible implications in relation to the relativistic models.

1. Introduction

Fifty years after the first atomic clock, it is no surprise that Terrestrial Time (TT) is at present derived from atomic clocks. International Atomic Time (TAI) is based on more than 250 atomic clocks distributed worldwide that provide its stability, whereas a small number of primary frequency standards provide its accuracy. Universal Coordinated Time (UTC), which is the basis of all legal time scales, is derived from TAI. To allow the construction of TAI and the general dissemination of time, clocks separated by thousands of kilometres must be compared and synchronized. This may be achieved with an uncertainty that can reach one nanosecond, using radio-transmission techniques such as the Global Positioning System (GPS) or two-way time transfer (TWTT) with geostationary satellites. Section 2 briefly presents the atomic time scales TAI and UTC and the time comparison techniques.

The achieved performances of atomic clocks and time transfer techniques imply that the definition of time scales and the clock comparison procedures must be considered within the framework of general relativity. In this framework, TAI is a realization of a time coordinate in the geocentric reference system and time comparisons are realized by coordinate synchronization in this system. The present relativistic modelling, presented in section 3, is perfectly adequate for the present needs of clocks and time transfer techniques. But, in the future a new generation of clocks may provide—in the laboratory or on board satellites—a relative uncertainty in frequency of 10^{-17} or less and new time transfer techniques will need to be developed to compare the new clocks. Section 4 reviews some implications of the upcoming developments.

2. Atomic time scales and time comparison methods

2.1. Atomic time scales: TAI and UTC

TAI was defined in 1970 by the International Committee for Weights and Measures as 'the time reference established by the BIH on the basis of the readings of atomic clocks operating in various establishments in accordance with the definition of the second'. In 1988 the responsibility of establishing TAI was transferred to the International Bureau for Weights and Measures (BIPM) in Sèvres (France).

As a deferred-time scale, TAI is computed each month using data provided by about 50 laboratories worldwide. Timing data from about 250 clocks are combined using an algorithm named ALGOS, resulting in an ensemble time scale named EAL. The stability of EAL is, by its construction, optimized for an averaging time around one month, and this is estimated in 2004 to be about 0.6×10^{-15}. Since EAL is free-running and is based on clocks that do not aim at realizing the SI second, its scale unit (the 'EAL second') may be different from the goal set in the definition of TAI. The rate of EAL is measured by comparison with a number of primary frequency standards which aim at realizing the SI second, and TAI is then derived from EAL by applying a rate correction so that the scale unit of TAI agrees with its definition (see section 3.4). The relative rate difference between EAL and TAI is presently (January 2005) 6.895×10^{-13} and may

0026-1394/05/030138+07$30.00

change slightly over time, as needed to ensure the accuracy of TAI. Over recent years, about ten primary standards have contributed occasionally or regularly to the accuracy of TAI, which is typically below 1×10^{-14} but is known to within $\pm 0.2 \times 10^{-14}$.

It has been known since the beginning of the 20th century that the Earth's rotation rate is irregular. Nevertheless, a time based on the rotation of the Earth taking exactly 24 h (Universal Time UT1) had been used for decades, and regulatory bodies tried in the 1960s to adapt the new atomic time to the former scale. This yielded in 1972 the definition of UTC: it differs from TAI by an integral number of seconds so that the difference between UT1 and UTC remains always smaller than 0.9 s. To ensure this, the International Earth Rotation and Reference Systems Service issues predictions of UT1−UTC and decides, when appropriate, to insert one leap second in UTC, or alternatively to remove one second from UTC. From 10 s in 1972, the difference between TAI and UTC has increased to 32 s in 1999, as a leap second has been inserted every 18 months on average. No further leap seconds have been necessary since 1999. UTC now serves as the basis for all legal time scales. See Arias and Guinot (2004) for more developments on the UTC system.

2.2. Time comparison methods

There are usually two ways to compare distant clocks: one method is to transport a portable clock from one clock to the other, properly accounting for the coordinate time accumulated during the transport (coordinate time differs from the proper time measured by the portable clock, see section 3.1); another method is to send an electromagnetic signal from one clock to the other, properly accounting for the coordinate time of propagation of the signal. Such procedures have been carried out since the development of atomic clocks.

The clock transport technique is rather cumbersome but has been much used until the development of the GPS in the 1980s. It then provided an uncertainty of some tens of nanoseconds, mainly limited by the instability of the transported clock. Although rarely used at present, an uncertainty of about a few nanoseconds is attained (Davis and Steele 1997), still limited by the instability of the transported clock. Eventually, when the frequency stability of transportable clocks reaches the order of 10^{-17}, an uncertainty of about a few picoseconds may be reached, which would be a challenge for radio-transmission techniques.

All commonly used techniques for clock comparison, described later, now rely on the transmission of electromagnetic signals. They can provide synchronization with an overall uncertainty of 1 ns or slightly less, e.g. for TWTT (Piester 2004), and a time stability of a few tens of picoseconds within measuring times of a few minutes to a few hours, e.g. for GPS carrier phase (Dach *et al* 2005). Therefore, frequency comparison to within the present clock performances (10^{-15}) can be achieved in a day or less.

The first important technique uses signals from navigation satellites, such as the GPS, the Russian GLONASS or the future European GALILEO system, to compare the local clock with the clock on board the satellite. In such techniques, each station receives signals from a number of satellites. A pseudo-random code stamped by the satellite clock is compared with a locally generated code stamped by the local clock. For GPS, codes are modulated at a 1 Mbps (C/A transmitted at 1.6 GHz) or 10 Mbps (P1/P2 transmitted at 1.6/1.2 GHz). In addition, the phase of the transmitted signals can also be measured (to within an unknown number of cycles). The comparison between the local and satellite clocks is obtained by subtracting the coordinate time of propagation of the signal (see section 3.5.2) from the measured time difference between the codes. The standard deviation is typically below 1 ns when averaging one hour of code measurements. Measurements of the signal carrier phase are 100 times more precise, but can only be used for frequency comparison.

The second important technique is TWTT in which signals are sent in both directions over the transmission path. The link may be a radio signal through geostationary communication satellites or a laser pulse through dedicated equipment on board satellites or optical fibres. In all cases, a two-way technique takes advantage of the reciprocity of the path to cancel or reduce some sources of error so that the measurement uncertainty is, in principle, somewhat reduced. In the TWTT technique currently used by time laboratories (Kirchner 1991), two stations simultaneously transmit a signal to a geostationary satellite. A transponder on board the satellite retransmits the signals for reception by the stations. In current systems, a pseudo-random code stamped by the local clock is modulated at a few megabytes per second. Transmissions are done in the Ku band (11 GHz to 14 GHz) using commercial communications satellites. The comparison between the two local clocks is obtained by proper combination of the measured time differences between the codes and the coordinate time of propagation of the signal (see section 3.5.3). The standard deviation of one 2-min measurement is typically a few hundred picoseconds or below.

3. Relativistic time comparisons

3.1. Introduction

In relativity it turns out to be useful to distinguish locally measurable quantities from coordinates that are, by definition, dependent on conventions. One generally speaks of proper and coordinate quantities:

(i) Proper quantities are the direct results of observation without involving any information that is dependent on conventions (such as, e.g., the choice of a space–time reference frame or a convention of synchronization). For metrology the most fundamental quantities are proper time and proper length, measured by a particular observer (proper to that observer). This also includes quantities that are not the real, physical results of observations, but correspond, in principle, to proper quantities (e.g. the proper time of an observer placed at the geocentre or infinitely far from the solar system).

(ii) Coordinate quantities are dependent on conventional choices, e.g. of a space–time coordinate system, a convention of synchronization etc. Examples are the coordinate time difference between two events (the difference between the time coordinates of these events) or the rate of a clock with respect to the coordinate time of some space–time reference system, which are both dependent on the chosen reference system.

For time metrology the most fundamental quantities are the proper time of a clock (the physical, local output of an ideal clock) and the coordinate time of a conventional space–time reference system (as defined, e.g., by the IAU, see section 3.3). For example, a Cs primary frequency standard produces a realization of the proper time at its location, but TAI is a realization of TT, a time coordinate defined by the IAU (see section 3.4).

Owing to the curvature of space–time, the scale units of space–time coordinate systems have, in general, no globally constant relation to proper quantities. In the framework of Newtonian mechanics (using Euclidean geometry) it is always possible to define coordinates in such a way that their scale units are equal to proper quantities everywhere, and it is therefore not necessary to explicitly distinguish between them. This is impossible in general relativity, where the relation between proper quantities and coordinate scale units is dependent on the position in space–time of the measuring observer. For metrology this implies, for example, that the relationship between a coordinate time interval and a measured proper time interval is dependent on the position of the measuring clock.

3.2. Simultaneity and synchronization

In the context of relativity theory, there is no *a priori* definition of simultaneity of two distant events and therefore synchronization or, more generally, comparison of distant clocks is arbitrary. It becomes subject to a conventional choice, called a convention of simultaneity and synchronization. The first such convention was proposed by Einstein (1905), and uses the exchange of electromagnetic signals, the so called 'Einstein synchronization convention'. Other conventions are, for example, 'slow clock transport synchronization' or 'coordinate synchronization'. The latter is the natural choice when the clock comparisons are to serve the purpose of realizing coordinate time scales (e.g. the construction of TAI). It is defined as (see e.g. Klioner (1992)):

> 'Two events fixed in some reference system by the values of their coordinates (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2) are considered to be simultaneous with respect to this reference system, if the values of time coordinate corresponding to them are equal: $t_1 = t_2$. This definition of simultaneity (and corresponding definition of synchronization) we shall call coordinate simultaneity (and coordinate synchronization).'

Clearly, synchronization by this definition is entirely dependent on the chosen space–time coordinate system, which needs to be defined consistently in a relativistic framework. Such definitions were provided by IAU resolutions in 1991 (IAU 1992, McCarthy 1992) and in 2000 (IAU 2001, McCarthy and Petit 2004), the relevant parts of which are summarized in the next sections.

3.3. Relativistic definitions of space–time coordinate systems

In order to describe the time and frequency comparison observations, one has to first choose the proper relativistic reference systems (Soffel *et al* 2003) best suited to the problem at hand. The barycentric celestial reference system (BCRS) should be used for all experiments not confined to the vicinity of the Earth, while the geocentric celestial reference system (GCRS) is physically adequate to describe processes occurring in the vicinity of the Earth. These systems were first defined by the IAU Resolution A4 (1991) which contains nine recommendations, the first four of which are relevant to our present discussion.

In the first recommendation, the metric tensor for space–time coordinate systems $(t, \boldsymbol{x})$ centred at the barycentre of an ensemble of masses is recommended in the form

$$\begin{aligned} g_{00} &= -1 + \frac{2U(t,\boldsymbol{x})}{c^2} + \mathcal{O}(c^{-4}),\\ g_{0i} &= \mathcal{O}(c^{-3}),\\ g_{ij} &= \delta_{ij}\left(1 + \frac{2U(t,\boldsymbol{x})}{c^2}\right) + \mathcal{O}(c^{-4}), \end{aligned} \tag{1}$$

where c is the speed of light in vacuum ($c = 299\,792\,458$ m s^{-1}) and U is the Newtonian gravitational potential (here a sum of the gravitational potentials of the ensemble of masses, and of an external potential generated by bodies external to the ensemble, the latter potential vanishing at the origin). The recommended form of the metric tensor can be used, not only to describe the barycentric reference system of the whole solar system, but also to define the geocentric reference system centred on the centre of mass of the Earth with U now depending upon geocentric coordinates.

In the second recommendation, the origin and orientation of the spatial coordinate grids for the barycentric and geocentric reference systems are defined.

The third recommendation defines Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG) as the time coordinates of the BCRS and GCRS, respectively, and, in the fourth recommendation, another time coordinate named TT, is defined for the GCRS.

In the following years, it became obvious that this set of recommendations was not sufficient, especially with respect to planned astrometric missions with accuracies of microarcseconds (GAIA and SIM) and with respect to the expected improvement of atomic clocks and the planned space missions involving such clocks and improved time transfer techniques (ACES, see section 4.1). For that reason the IAU working group 'Relativity for astrometry and celestial mechanics' together with the BIPM–IAU Joint Committee for relativity suggested an extended set of Resolutions that was finally adopted at the IAU General Assembly in Manchester in the year 2000 as Resolutions B1.3 to B1.5 and B1.9.

Resolution B1.3 concerns the definition of BCRS and GCRS. The Resolution recommends to write the metric tensor of the BCRS in the form

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}),\\ g_{0i} &= -\frac{4}{c^3}w^i + \mathcal{O}(c^{-5}),\\ g_{ij} &= \delta_{ij}\left(1 + \frac{2w}{c^2}\right) + \mathcal{O}(c^{-4}), \end{aligned} \tag{2}$$

where w is a scalar potential and w^i a vector potential. This extends the form of the metric tensor given in the IAU 1991

Resolutions, so that its accuracy is now sufficient for all time and frequency applications foreseen in the next few years. For the GCRS, Resolution B1.3 also adds that the spatial coordinates are kinematically non-rotating with respect to the barycentric ones.

Resolution B1.4 provides the form of the expansion of the post-Newtonian potential of the Earth to be used with the metric of Resolution B1.3.

Resolution B1.5 applies the formalism of Resolutions B1.3 and B1.4 to the problems of time transformations and realization of coordinate times in the solar system. Resolution B1.5 is based upon a mass monopole spin dipole model. It provides an uncertainty not larger than 5×10^{-18} in rate and, for quasi-periodic terms, not larger than 5×10^{-18} in rate amplitude and 0.2 ps in phase amplitude, for locations farther than a few solar radii from the Sun. The same uncertainty also applies to the transformation between TCB and TCG for locations within 50 000 km of the Earth (see section 3.6).

Resolution B1.9 concerns the definition of TT and will be developed in the next section.

3.4. *Time scales in the solar system*

As mentioned above, TCB and TCG are the time coordinates of the BCRS and GCRS, respectively. The IAU 1991 Recommendation 3 defined the scale unit of TCB and TCG to be consistent with the SI second. This means that if readings of proper time of an observer, expressed in SI seconds, are recomputed into TCB or TCG using the formulae from the IAU Resolutions, without any additonal scaling, one gets corresponding values of TCB or TCG in the intended units. It also defines the origin of TCB and TCG by the following relation to TAI:

TCB (resp. TCG) = TAI + 32.184 s on 1 January 1977,
0 h TAI, at the geocentre.

TT is another coordinate time for the GCRS, differing from TCG by a constant rate.

$$\frac{\mathrm{d(TT)}}{\mathrm{d(TCG)}} = 1 - L_G.$$

In the original definition (IAU 1991 Recommendation 4), this rate was chosen such as the scale unit of TT be consistent with the SI second on the rotating geoid, i.e. $L_G = U_g/c^2$, where U_g is the gravity (gravitational + rotational) potential on the geoid. Some shortcomings appeared in this definition of TT when considering accuracies below 10^{-17} because of the uncertainties in the realization of the geoid at the level required. Therefore it was decided to dissociate the definition of TT from the geoid while maintaining continuity with the previous definition. Resolution B1.9 (2000) turned L_g into a defining constant with its value fixed to $6.969\,290\,134\times10^{-10}$. The origin of TT is defined, so that TCB and TCG coincide with TT in origin, by

$$\mathrm{TT} = \mathrm{TAI} + 32.184\,\mathrm{s\ on\ 1\ January\ 1977,\ 0\,h\ TAI}.$$

TT is a theoretical time scale and can have different realizations that are differentiated by the notation TT(realization). TAI can provide one of them: TT(TAI) = TAI + 32.184 s. Because TAI is computed in 'real-time' every month and has operational constraints (e.g. no correction for a mistake discovered many days after the publication), it does not provide an optimal realization of TT. The BIPM therefore computes in post-processing another realization TT(BIPMxxxx), where 'xxxx' is the year of computation, which is based on EAL and on a weighted average of the primary frequency standards data (Guinot 1988, Petit 2003). Several versions have been computed, the latest of which is TT(BIPM2004) (see ftp://62.161.69.5/pub/tai/scale/), with an estimated accuracy below 2×10^{-15}.

3.5. *Relativistic theory for time comparisons in the vicinity of the Earth (GCRS)*

We present here formulae or references that allow one to perform time transfer and synchronization in the vicinity of the Earth (typically up to geosynchronous orbit or slightly above). Evaluating the contributions of the higher order terms in the metric of the geocentric reference system (Resolution B1.3), it is found that the IAU 1991 framework with the metric of the form (1) is sufficient for time and frequency applications in the GCRS in the light of present and foreseeable future clock accuracies. Nevertheless, in applying the IAU 1991 formalism, some care needs to be taken when evaluating the Earth's potential at the location of the clock, especially when accuracy of order 10^{-18} is required (Klioner 1992, Wolf and Petit 1995, Petit and Wolf 1997).

In this framework, the proper time of a clock A located at the GCRS coordinate position $\boldsymbol{x}_A(t)$, and moving with the coordinate velocity $\boldsymbol{v}_A = \mathrm{d}\boldsymbol{x}_A/\mathrm{d}t$, is

$$\frac{\mathrm{d}\tau_A}{\mathrm{d}t} = 1 - \frac{1}{c^2}\left[\frac{\boldsymbol{v}_A^2}{2} + U_E(\boldsymbol{x}_A) + V(\mathcal{X}_A) - V(\mathcal{X}_E) - x_A^i \partial_i V(\mathcal{X}_E)\right]. \tag{3}$$

Here, U_E denotes the Newtonian potential of the Earth at the position $\boldsymbol{x}_A$ of the clock in the GRS frame, and V is the sum of the Newtonian potentials of the other bodies (mainly the Sun and the Moon) computed at a location $\mathcal{X}$ in barycentric coordinates, either at the position $\mathcal{X}_E$ of the Earth's centre of mass, or at the clock location $\mathcal{X}_A$. Only terms required for frequency transfer with uncertainty of order 10^{-18} have been kept. For application to any experiment at a level of uncertainty greater than 5×10^{-17} on Earth or on board a satellite in low Earth orbit, one can keep only the first three terms in the relation (3) between the proper time τ_A and the coordinate time t:

$$\frac{\mathrm{d}\tau_A}{\mathrm{d}t} = 1 - \frac{1}{c^2}\left[\frac{\boldsymbol{v}_A^2}{2} + U_E(\boldsymbol{x}_A)\right]. \tag{4}$$

The relativistic treatment of time transfer and clock synchronization results in the integration of a differential relation, according to the problem at hand. To compute an interval of coordinate time from a measured interval of proper time (e.g. clock transportation), one has to integrate (4), whereas to compute the coordinate time of propagation of a light signal, one has to first solve the metric equations ((1) or (2)) with $\mathrm{d}s^2 = 0$ and integrate the solution obtained. These operations are performed in the following sections in order to provide uncertainty of 1 ps or below in time comparisons.

Note that, in section 3.5.1, the integration may provide larger uncertainty owing to uncertainties in the path of integration or in the velocity to be integrated.

3.5.1. Clock transportation. When synchronizing clocks A and B by transporting a clock C from A to B, one needs to compute the coordinate time elapsed between the two events 'comparison of clocks A and C' and 'comparison of clocks B and C'. This is obtained from the proper time measured by clock C by integrating equation (4) along the trajectory of clock C. One obtains

$$\Delta t = \int_{\mathrm{A}}^{\mathrm{B}} \left(1 + \frac{1}{c^2}\left(U + \frac{v^2}{2}\right)\right) \mathrm{d}\tau. \tag{5}$$

Note that the integral is carried out in the GCRS which is not rotating with the Earth and that t is TCG. One usually works in a frame of reference rotating with the Earth, with TT as a coordinate time. One then obtains

$$\Delta \mathrm{TT} = \int_{\mathrm{A}}^{\mathrm{B}} \left(1 + \frac{1}{c^2}\left(U + \frac{(\omega \times r)^2 + v'^2}{2}\right) - L_G\right) \mathrm{d}\tau + \frac{1}{c^2}\int_{\mathrm{A}}^{\mathrm{B}} (\omega \times r)\cdot v' \,\mathrm{d}\tau, \tag{6}$$

where r and v' are the position vector and velocity of the clock in the rotating frame and where ω is the rotation vector of the Earth. Under the first integral are the gravitational redshift and time dilation terms, and the second integral represents the Sagnac effect.

As an example, Davis and Steele (1997) carried a clock on an aircraft over a route between London and Washington DC. The proper time interval recorded by the flying clock is denoted $\Delta\tau$ and the interval of coordinate time TT is denoted Δt. The difference $\Delta\tau - \Delta t$ was +28.4 ns westward (resp. +24.6 ns eastward) owing to the gravitational redshift and −8.0 ns (resp. −8.1 ns) owing to time dilation. For the westward flight, the Sagnac effect is +19.8 ns, so the flying clock gains 40.2 ns relative to the coordinate time. On the other hand, for the eastward flight from Washington DC to London, the Sagnac effect is −17.1 ns and the flying clock loses 0.6 ns.

3.5.2. One-way time transfer. This section is based on the study by Blanchet *et al* (2001, section 3.1) (see also Petit and Wolf (1994)).

Let A be the emitting station, with GCRS position $\boldsymbol{x}_{\mathrm{A}}(t)$, and B the receiving station, with position $\boldsymbol{x}_{\mathrm{B}}(t)$. We use t = TCG and the calculated coordinate time intervals are in TCG. The corresponding time intervals in TT are obtained by multiplying with $(1 - L_G)$. We denote by t_{A} the coordinate time at the instant of emission of a light signal, and by t_{B} the coordinate time at the instant of reception. We put $r_{\mathrm{A}} = |\boldsymbol{x}_{\mathrm{A}}(t_{\mathrm{A}})|$, $r_{\mathrm{B}} = |\boldsymbol{x}_{\mathrm{B}}(t_{\mathrm{B}})|$ and $R_{\mathrm{AB}} = |\boldsymbol{x}_{\mathrm{B}}(t_{\mathrm{B}}) - \boldsymbol{x}_{\mathrm{A}}(t_{\mathrm{A}})|$. Up to the order $1/c^3$ the coordinate time of signal propagation $T_{\mathrm{AB}} \equiv t_{\mathrm{B}} - t_{\mathrm{A}}$ is given by

$$T_{\mathrm{AB}} = \frac{R_{\mathrm{AB}}}{c} + \frac{2GM_{\mathrm{E}}}{c^3}\ln\left(\frac{r_{\mathrm{A}} + r_{\mathrm{B}} + R_{\mathrm{AB}}}{r_{\mathrm{A}} + r_{\mathrm{B}} - R_{\mathrm{AB}}}\right), \tag{7}$$

where GM_{E} is the geocentric gravitational constant and where the logarithmic term represents the Shapiro time delay. Between a low Earth orbit satellite (LEO) and the ground, the Shapiro time delay is a few picoseconds. For a GPS or geostationary satellite, it is a few tens of picoseconds.

In a real experiment, the position of the receptor B is known at the time of emission t_{A} rather than at the time of reception t_{B}, i.e. we have access to $\boldsymbol{x}_{\mathrm{B}}(t_{\mathrm{A}})$ rather than $\boldsymbol{x}_{\mathrm{B}}(t_{\mathrm{B}})$, and the formula (7) gets modified by Sagnac correction terms consistently to the order $1/c^3$. In this case the formula becomes

$$\begin{aligned} T_{\mathrm{AB}} = {} & \frac{D_{\mathrm{AB}}}{c} + \frac{\boldsymbol{D}_{\mathrm{AB}}\cdot \boldsymbol{v}_{\mathrm{B}}(t_{\mathrm{A}})}{c^2} \\ & + \frac{D_{\mathrm{AB}}}{2c^3}\left(\boldsymbol{v}_{\mathrm{B}}^2 + \frac{(\boldsymbol{D}_{\mathrm{AB}}\cdot \boldsymbol{v}_{\mathrm{B}})^2}{D_{\mathrm{AB}}^2} + \boldsymbol{D}_{\mathrm{AB}}\cdot \boldsymbol{a}_{\mathrm{B}}\right) \\ & + \frac{2GM_{\mathrm{E}}}{c^3}\ln\left(\frac{r_{\mathrm{A}} + r_{\mathrm{B}} + D_{\mathrm{AB}}}{r_{\mathrm{A}} + r_{\mathrm{B}} - D_{\mathrm{AB}}}\right), \end{aligned} \tag{8}$$

where $\boldsymbol{D}_{\mathrm{AB}} = \boldsymbol{x}_{\mathrm{B}}(t_{\mathrm{A}}) - \boldsymbol{x}_{\mathrm{A}}(t_{\mathrm{A}})$ and $D_{\mathrm{AB}} = |\boldsymbol{D}_{\mathrm{AB}}|$, where $\boldsymbol{v}_{\mathrm{B}}(t_{\mathrm{A}})$ denotes the coordinate velocity of the station B at that instant and where $\boldsymbol{a}_{\mathrm{B}}$ is the acceleration of B. The second term in (8) represents the Sagnac term of order $1/c^2$ and can amount to up to 200 ns for LEOs and 133 ns for GPS. The third term, or Sagnac term of order $1/c^3$, is a few picoseconds (up to 10 ps for a geostationary satellite). The fourth term is the Shapiro delay, discussed earlier.

3.5.3. TWTT using artificial satellites. We distinguish two types of TWTT. In the first one, the signal transmission is between the two clocks A and B which are to be compared with A on board an artificial satellite. This case is described by Blanchet *et al* (2001, section 3.2). One signal is emitted from A at instant t_{A} and received at B at instant t_{B} and the other signal is emitted from B at instant t'_{B} and received at A at instant t'_{A}. The intervals of time between emission and reception at A and B, $t_{\mathrm{AA'}} = t'_{\mathrm{A}} - t_{\mathrm{A}}$ and $t_{\mathrm{B'B}} = t_{\mathrm{B}} - t'_{\mathrm{B}}$, are measured. The quantity required for synchronization derives from the application of the one-way time transfer formulae (preceding section) to the two signal transmissions involved.

This TWTT technique may be performed for two different stations B and C, thereby allowing time transfer between B and C, if clock A on board the satellite measures, in addition, the time interval between its transmission to B and to C. This type of time transfer is applied in the particular case of the LASSO method, where in addition $t_{\mathrm{A}} = t'_{\mathrm{A}}$. The relativistic treatment of the LASSO clock synchronization is described by Petit and Wolf (1994, section 4).

In the second type, two clocks A and B (usually on the ground) are compared via an artificial satellite S. Two signals are transmitted from A and from B to S, where each one is immediately retransmitted towards the other station. The clocks at A and B measure the time interval between emission and reception. This is the TWTT technique routinely used by time laboratories for clock comparisons. The relativistic treatment of TWTT is described by Petit and Wolf (1994, section 3) for the case of A being a geostationary satellite and by Klioner and Fukushima (1994) for an arbitrary satellite. The quantity required for synchronization derives from the application of the one-way time transfer formulae (preceding section) to the four signal transmissions involved.

3.6. Relativistic theory for time transformations in the solar system (BCRS)

Following Resolutions B1.3(2000) and B1.4(2000), the metric tensor in the BCRS is expressed as

$$\begin{aligned} g_{00} &= -\left(1 - \frac{2}{c^2}(w_0(t,\boldsymbol{x}) + w_L(t,\boldsymbol{x})) + \frac{2}{c^4}(w_0^2(t,\boldsymbol{x}) + \Delta(t,\boldsymbol{x}))\right) \\ g_{0i} &= -\frac{4}{c^3} w^i(t,\boldsymbol{x}), \\ g_{ij} &= \left(1 + \frac{2w_0(t,\boldsymbol{x})}{c^2}\right)\delta_{ij}, \end{aligned} \tag{9}$$

where $(t \equiv \mathrm{TCB}, \boldsymbol{x})$ are the barycentric coordinates, $w_0 = G\sum_{\mathrm{A}} M_{\mathrm{A}}/r_{\mathrm{A}}$, with the summation carried out over all solar system bodies A, $\boldsymbol{r}_{\mathrm{A}} = \boldsymbol{x} - \boldsymbol{x}_{\mathrm{A}}$, $r_{\mathrm{A}} = |\boldsymbol{r}_{\mathrm{A}}|$ and where w_L contains the expansion in terms of multipole moments, as defined in Resolution B1.4(2000) and references therein, required for each body. In many cases the mass-monopole approximation ($w_L = 0$) may be sufficient to reach the above mentioned uncertainties but this term should be kept to ensure the consistency in all cases. The values of masses and multipole moments to be used may be found in IAU or IERS documents (McCarthy and Petit 2004), but care must be taken that the values are in SI units (not in the so-called TDB units or TT units). The vector potential $w^i(t,\boldsymbol{x}) = \sum_{\mathrm{A}} w^i_{\mathrm{A}}(t,\boldsymbol{x})$ and the function $\Delta(t,\boldsymbol{x}) = \sum_{\mathrm{A}} \Delta_{\mathrm{A}}(t,\boldsymbol{x})$ can be computed from the expressions given in the IAU Resolution B1.5 (2000).

From (9) the transformation between proper time and TCB may be derived. It reads

$$\frac{\mathrm{d}\tau}{\mathrm{dTCB}} = 1 - \frac{1}{c^2}\left(w_0 + w_L + \frac{v^2}{2}\right) + \frac{1}{c^4}\left(-\frac{1}{8}v^4 - \frac{3}{2}v^2 w_0 + 4v^i w^i + \frac{1}{2}w_0^2 + \Delta\right). \tag{10}$$

Evaluating the Δ_{A} terms for all bodies of the solar system, we find that $|\Delta_{\mathrm{A}}(t,\boldsymbol{x})|/c^4$ may reach at most a few parts in 10^{17} in the vicinity of Jupiter and about 1×10^{-17} close to the Earth. Presently, however, for all planets except the Earth, the magnitude of $\Delta_{\mathrm{A}}(t,\boldsymbol{x})/c^4$ in the vicinity of the planet is smaller than the uncertainty originating from its mass or multipole moments so that it is practically not needed to account for these terms. Nevertheless, when new astrometric observations allow to derive the mass and moments with adequate uncertainty, it will be necessary to do so. In any case, for the vicinity of a given body A, only the effect of $\Delta_{\mathrm{A}}(t,\boldsymbol{x})$ is needed in practice for our accuracy specifications. For a clock in the vicinity of the Earth, to be compared with other clocks in the solar system or to TCB, it may thus be needed to account for $\Delta_{\mathrm{E}}(t,\boldsymbol{x})/c^4$.

Similarly, the transformation between TCB and TCG may be written as

$$\begin{aligned} \mathrm{TCB} = \mathrm{TCG} &+ c^{-2}\left[\int_{t_0}^{t}\left(\frac{v_{\mathrm{E}}^2}{2} + w_{0\mathrm{ext}}(\boldsymbol{x}_{\mathrm{E}})\right)\mathrm{d}t + v_{\mathrm{E}}^i r_{\mathrm{E}}^i\right] \\ &- c^{-4}\left[\int_{t_0}^{t}\left(-\frac{1}{8}v_{\mathrm{E}}^4 - \frac{3}{2}v_{\mathrm{E}}^2 w_{0\mathrm{ext}}(\boldsymbol{x}_{\mathrm{E}}) + 4v_{\mathrm{E}}^i w_{\mathrm{ext}}^i(\boldsymbol{x}_{\mathrm{E}}) + \frac{1}{2}w_{0\mathrm{ext}}^2(\boldsymbol{x}_{\mathrm{E}})\right)\mathrm{d}t - \left(3w_{0\mathrm{ext}}(\boldsymbol{x}_{\mathrm{E}}) + \frac{v_{\mathrm{E}}^2}{2}\right)v_{\mathrm{E}}^i r_{\mathrm{E}}^i\right], \end{aligned} \tag{11}$$

where t is TCB and where the index 'ext' refers to all bodies except the Earth. This equation is composed of terms evaluated at the geocentre (the two integrals) and of position-dependent terms in r_{E}, with position-dependent terms in higher powers of r_{E} having been found to be negligible. Terms in the second integral of (11) are secular and quasi-periodic. They amount to $\sim 1.1 \times 10^{-16}$ in rate (dTCB/dTCG) and primarily a yearly term of $\sim$30 ps in amplitude (i.e. corresponding to periodic rate variations of amplitude $\sim 6 \times 10^{-18}$). Terms in $\Delta_{\mathrm{ext}}(t,\boldsymbol{x})$ that would appear in this integral are negligible in the vicinity of the Earth where this formula is to be used. Besides the position-dependent terms in c^{-2}, which may reach several microseconds, position-dependent terms in c^{-4} (the last two terms in (11)) are not negligible and reach, for example, an amplitude of 0.4 ps ($\sim 3 \times 10^{-17}$ in rate) in geostationary orbit.

4. Conclusions and future prospects

The relativistic theory presented in this paper and in the references allows one to perform all clock comparisons and the realization of coordinate times with an uncertainty well below the present performances of atomic clocks. However, these performances are steadily improving. Progress with Cs clocks may be limited to 10^{-16} accuracy because of the effect of collisions between atoms but transitions in other atoms and ions are promising. A rubidium fountain is already operating at the BNM/SYRTE in Paris and numerous other projects are underway in the world, see a recent review in Bauch and Telle (2002). Below we examine some consequences of such improvements, in relation with the relativistic theory.

4.1. ACES-MWL and T2L2

The Atomic Clock Ensemble in Space (ACES) is a CNES/ESA mission scheduled for launch in early 2009, which consists of two atomic clocks and a time transfer system installed on board the International Space Station (ISS) (Salomon *et al* 2001, Bize *et al* 2004). The two clocks are a hydrogen maser from the Observatoire de Neuchâtel and a Cs cold atom clock funded by CNES (Centre National d'Etudes Spatiales). The clocks are linked to the ground via a microwave link (MWL) based on present TWTT technology, but with significantly improved precision. The objectives of ACES are (i) to explore and demonstrate the high performances of the cold atom space clock, (ii) to achieve time and frequency transfer with stability better than 10^{-16} and (iii) to perform fundamental physics tests. A detailed account can be found in Salomon *et al* (2001). In the initial mission design a second time transfer system using optical frequencies in a two-way configuration (Time Transfer by Laser Light, T2L2) was planned, but it had to be abandoned because of financial difficulties.

Both time transfer methods (MWL and T2L2) are designed for precisions below 1 ps at an integration time of $\approx$300 s (duration of an ISS passage above a ground station), and of a few (<10) picoseconds at an integration time of one day (maximum time interval between two successive passes of the ISS above a ground station). This allows frequency comparisons between the space and ground clocks with stabilities of $\approx 10^{-15}$ over one passage and 10^{-16} or better over one day, which is required for the objectives of the mission.

To achieve such precision the ACES-MWL uses three frequencies (13.5 GHz uplink, 14.7 GHz and 2.2 GHz downlinks) with 100 Mbps pseudo-random codes stamped onto the 13.5 GHz and 14.7 GHz carriers and a 1 Mbps pseudo-random code onto the 2.2 GHz carrier. The two downlink frequencies allow for the cancellation of the ionospheric effect. Tropospheric, orbitography and instrumental effects are partially cancelled by the two-way architecture. Additionally, the MWL will measure the carrier phase of the signals and remove the cycle ambiguities using the code measurements (similarly to what is done in GPS).

The improved precision levels and the relatively large coordinate velocity of the ISS ($\approx$7700 m s^{-1} compared to $\approx$3900 m s^{-1} for GPS) require that particular care has to be taken in the relativistic modelling of the MWL time transfer. Recent work on that subject shows that the basic principles and approximations used in section 3.5 are still sufficient. However, some problems, particular to this type of link, need to be addressed, for example additional terms arising from the movement of the satellite between reception of the upward signal and emission of the downward signal (Sagnac type terms), or additional terms in the ionospheric corrections arising from the movement of the satellite between the emissions of the two downward signals. For the MWL such terms are not negligible with respect to the planned precision, and need to be modelled correctly in a rigorously relativistic framework.

4.2. Clocks and geophysics

To compare primary frequency standards, the chosen convention is to compare each one to a coordinate time by using equations (3) or (4) above. For a clock on Earth, this implies estimating the gravity (gravitational + rotational) potential at its location. We have estimated (Petit and Wolf 1997) that this may limit frequency comparisons for clocks on Earth to the level of 10^{-17} and this is what has been recently achieved (Pavlis and Weiss 2003) for the Cs fountain NIST-F1 in Boulder, Colorado.

It is possible that, when clocks accurate to parts in 10^{17} and below exist, they provide information on the Earth's gravitational potential, effectively becoming devices to measure it. If we extrapolate the recent pace of development in atomic clocks, this could happen in 10/15 years. To overcome this limitation, some of these ultra-accurate clocks should be placed in space to provide the reference against which the Earth clocks would be compared. First steps in this direction should happen within a few years with space clock projects aboard the ISS (see previous section). At the same time, geopotential models will improve too, thanks to space missions like GRACE and GOCE. Thus, the confrontation between atomic physics and geophysics could take place in a few decades.

References

Arias E F and Guinot B 2004 Coordinated Universal Time UTC: historical background and perspectives *Proc. Journées Systèmes de références spatio-temporels (Paris)* at press

Bauch A and Telle H R 2002 *Rep. Prog. Phys.* **65** 789

Bize S *et al* 2004 Advances in atomic fountains *C. R. Acad. Sci. Physique* **5** 829–43

Blanchet L *et al* 2001 Relativistic theory for time and frequency transfer to order c^{-3} *Astron. Astrophys.* **370** 320

Dach *et al* 2005 Geodetic frequency transfer *Proc. 19th European Frequency and Time Forum (Neuchâtel)* at press

Davis J A and Steele J McA 1997 A caesium flying clock experiment between NPL and USNO *Proc. 11th EFTF (Neuchâtel)* p 306

Einstein A 1905 Zur Elektrodynamik bewegter Körper *Ann. Phys., Lpz.* **17** 891–921

Guinot B 1988 *Astron. Astrophys.* **192** 370

IAU Information Bulletin 1992 vol 67, p 7

IAU Information Bulletin 2001 vol 88, p 28

IAU Information Bulletin 2001 vol 89, p 4 (erratum) available at http://www.iau.org/Activities/publications/bulletin/

Kirchner D 1991 *Proc. IEEE* **79-7** 983–90

Klioner S A 1992 *Celest. Mech. Dyn. Astrom.* **53** 81

Klioner S A and Fukushima T 1994 Relativistic effects in two-way time transfer via artificial satellites using laser techniques *Manuscripta Geodaetica* **19** 294–9

McCarthy D (ed) 1992 *IERS Standards* IERS TN 13, Paris Observatory, Paris

McCarthy D and Petit G (ed) 2004 IERS Conventions (2003), IERS TN 32, Verlag des BKG

Pavlis N and Weiss M A 2003 *Metrologia* **40** 66

Petit G 2003 A new realization of Terrestrial Time *Proc. 35th PTTI Meeting*

Petit G and Wolf P 1994 Relativistic theory for picosecond time transfer in the vicinity of the Earth *Astron. Astrophys.* **286** 971

Petit G and Wolf P 1997 Computation of the relativistic rate shift of a frequency standard *IEEE Trans. Instrum. Meas.* **46** 201–4

Piester D 2004 Time transfer between USNO and PTB: operation and calibration results *Proc. 36th PTTI Meeting* at press

Salomon C *et al* 2001 Cold atoms in space and atomic clocks: ACES *C. R. Acad. Sci. Paris Série* 4, 1313–20

Soffel M *et al* 2003 The IAU 2000 resolutions for astrometry, celestial mechanics and metrology in the relativistic framework: explanatory supplement *Astron. J.* **126** 2687–706

Wolf P and Petit G 1995 Relativistic theory for clock syntonization and the realization of coordinate times *Astron. Astrophys.* **304** 653–61