# A NOVEL RANGE MIGRATION ALGORITHM OF GEO SAR ECHO DATA 

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## 1. INTRODUCTION

Synthetic Aperture Radar (SAR) has been developed remarkably since the invention in 50's last century. With the development of application requirements, the disadvantages of low earth orbit (LEO) SAR become more and more apparent, such as long revisit period and small coverage area etc. An effective solution to overcome these shortages is to heighten the satellite orbit to geosynchronous earth orbit (GEO).
GEO SAR operates on 36000 km orbit height to achieve a large coverage area, and its orbit period and revisit period are 1 day respectively. In 1978, the conception of GEO SAR was firstly introduced by K.Tomiyasu. A few years later, under the support of NASA, K.Tomiyasu ${ }^{[1]}$ improved the system firstly introduced, and analyzed the basic system parameters carefully. In 1987, Lesley M ${ }^{[2]}$ researched on some aspects of GEO SAR including conception, realization and so on. In 21 st century, much more attentions are paid on GEO SAR studies. Under the support of NASA, JPL made much research work and obtained important results in orbit design and its applications.
So far, there are few open literatures about imaging algorithm in GEO SAR. Due to large slant range, the spacevariance of Doppler parameters in GEO SAR becomes much worse than that in LEO SAR, and limits the use of classical imaging algorithms in GEO SAR. So, the imaging algorithm in GEO SAR is a challenging work to do. In this paper, a Modified Range Migration Algorithm (RMA) is proposed to overcome the space-variance of Doppler parameters by compensating the velocity change in different sub-scene. Simulation results fully verify the effectiveness of derived algorithm.
The rest parts are organized as follows. In the 2nd part, complete signal model of point target was established. In the 3rd part, modified RMA algorithm is derived and the focusing capability of the classical RMA is analyzed. In the 4th part, the listed simulation results validate the presented analysis. The conclusions are included in 5 th part.

## 2. SIGNAL MODEL

The 3-dimensional system topology of GEO SAR is shown in Fig 1a. In Fig 1a, GEO SAR platform is running in a curve trajectory. In the following discussion, we suppose that the real curve trajectory could be approximated as a linear trajectory using equivalent slant range model ${ }^{[3]}$. Based on the above mentioned assumption, we can simplify 3-D coordinate frame in Fig 1a as 2-D coordinate frame in Fig 1b, where platform flies along x-axis and $R_{0}$ is the central range of the scene.

As shown in Fig 1b, a point target with backscattering coefficient $\sigma$ locates at $(X, R)$. If the transmitted pulse


Fig 1 Coordinates system of GEO SAR system: a 3-D frame; b 2-D frame
is $p(t)$, during $n$-th PRT, the received echo in baseband can be expressed as

$$
\begin{equation*}
s\left(t, t_{n}\right)=\sigma p\left(t-2 R_{n} / c\right) \exp \left(-j 4 \pi R_{n} / \lambda\right) \tag{1}
\end{equation*}
$$

where $\lambda$ is the wavelength, and range $R_{n}$ can be calculated as

$$
\begin{equation*}
R\left(t_{n}\right)=\sqrt{R^{2}+\left(v_{t} t_{n}-X\right)^{2}} \tag{2}
\end{equation*}
$$

where $v_{t}$ is the equivalent velocity introduced by equivalent slant range model. In eq.(2), we propose a model of $v_{t}$ as the function of $(X, R)$, i.e.

$$
\begin{equation*}
v_{t}(X, R)=v_{0}+g(X, R) \tag{3}
\end{equation*}
$$

where $g\left(X_{0}, R_{0}\right)=0$ holds, and $\left(X_{0}, R_{0}\right)$ is the central of the scene. The model in eq.(3) implies that if a linear trajectory is adopted to approximate the curve one, the equivalent velocity varies with the location of the point target. In other words, the model in eq.(3) stands for space-variance of Doppler parameters.

## 3. IMAGING ALGORITHM

## 3.1 modified Range Migration Algorithm

With principle of stationary phase (PSP) the 2-D spectrum of the echo after range compression can be written as

$$
\begin{equation*}
S\left(k_{r}, k_{x}\right)=\sigma \exp \left(-j R \sqrt{k_{r}^{2}-k_{x}^{2}}-j k_{x} X\right) \tag{4}
\end{equation*}
$$

where $k_{r}, k_{x}$ denote range and azimuth wave-number, and can be defined as $k_{r}=4 \pi\left(f_{c}+f\right) / c, k_{x}=2 \pi f_{d} / v_{t}$. Note that the equivalent velocity $v_{t}$ is varying with the location of point target.

Considering the space-variance of Doppler parameters, we approximate the phase term in eq.(4) as follows:

$$
\begin{equation*}
\Phi\left(k_{r}, k_{x}\right)=-R \sqrt{k_{r}^{2}-k_{x}^{2}}-k_{x}^{2} X \approx-R \sqrt{k_{r}^{2}-k_{x 0}^{2}}-k_{x 0} \cdot \frac{v_{0}}{v_{t}} X-\frac{R k_{x 0}^{2}}{2 \sqrt{k_{r}^{2}-k_{x 0}^{2}}}\left(1-\left(\frac{v_{0}}{v_{t}}\right)^{2}\right) \tag{5}
\end{equation*}
$$

where $v_{0}$ is the equivalent velocity in the scene center, and the corresponding wave-number $k_{x 0}$ is $k_{x 0}=2 \pi f_{d} / v_{0}$. The third phase term in eq.(5) is the extra phase term introduced by space-variance of Doppler parameters.
Introducing STOLT variables transform, i.e. $k_{X}=k_{x 0} ; k_{R}=\sqrt{k_{r}^{2}-k_{x 0}^{2}}$, the phase term in eq.(5) can be rewritten as:

$$
\begin{equation*}
\Phi\left(k_{R}, k_{X}\right)=-R k_{R}-k_{X} \cdot \frac{v_{0}}{v_{t}} X-\frac{R k_{X}^{2}}{2 k_{R}}\left(1-\left(\frac{v_{0}}{v_{t}}\right)^{2}\right) \tag{6}
\end{equation*}
$$

The 3rd phase term is the additional phase introduced by the space variance of the Doppler parameters, and ignored in the classical RMA. Under the condition of large slant range or high azimuth resolution, the $3^{\text {rd }}$ phase


Table 1 simulation parameters of the GEO SAR

| wavelength | 0.094 m | bandwidth | 18 MHz |
| :---: | :---: | :---: | :---: |
| pulse <br> width | 20 us | sampling <br> rate | 22 MHz |
| PRF | 150 Hz | central range | 33000 km |

Fig 2 flowchart of modified RMA in GEO SAR
term will be too large to ignore in the imaging algorithm. In GEO SAR the slant range is 40000 km , so the $3^{\text {rd }}$ phase term can not be ignored. The classical RMA should be modified to compensate the $3^{\text {rd }}$ phase term. That is

$$
\begin{equation*}
\Phi_{c 2}\left(k_{R}, k_{X}\right)=\frac{R k_{X}^{2}}{2 k_{R}}\left(1-\left(\frac{v_{0}}{v_{t}}\right)^{2}\right) \tag{7}
\end{equation*}
$$

With phase compensation in eq.(6), eq.(4) can be rewritten as:

$$
\begin{equation*}
S\left(k_{R}, k_{X}\right)=\sigma \exp \left(-j R k_{R}-j k_{X} \cdot \frac{v_{0}}{v_{t}} X\right) \tag{8}
\end{equation*}
$$

Using 2-D IFT to eq.(8), the well-focused scene can be obtained.
The new phase compensation term in eq.(7) shows the idea that the modified RMA adopts to focus the large scene, i.e. sub-scene processing. Finally we can recombine the well-focused sub-scene into a large one.

### 3.2 The focusing capability of classical RMA

Eq.(7) is the 2 nd order residual phase term after classical RMA processing. In the GEO SAR system, supposing the support area of $k_{x}$ is $\left[4 \pi \sin \theta_{0} / \lambda-\pi / \rho_{a}, 4 \pi \sin \theta_{0} / \lambda+\pi / \rho_{a}\right]$, where $\rho_{a}$ is azimuth resolution; the maximum of the 2 nd order phase term can be rewritten as

$$
\begin{equation*}
|\Delta \Phi|_{\max } \approx \frac{R_{0}\left(\pi / \rho_{0}\right)^{2}}{8 \pi / \lambda}\left(1-\left(\frac{v_{0}}{v_{t}}\right)^{2}\right) \tag{9}
\end{equation*}
$$

In order to avoid defocusing, the maximum of the phase term in eq.(9) should be less than $\pi / 4$, i.e. $|\Delta \Phi|_{\max }<\pi / 4$. According to the model in eq.(3), it could be simplified as

$$
\begin{equation*}
g(X, R)<v_{0} / \sqrt{1-\frac{2 \rho_{a}^{2}}{\lambda R_{0}}}-v_{0} \tag{10}
\end{equation*}
$$

The flowchart of the modified RMA is shown in Fig 2. The extant of the sub-scene can be calculated by eq.(10).

## 4. SIMULATION RESULTS

The simulation parameters are listed in Table 1 and a large scene in Fig 3 near the equator is adopted. According to the parameters in Table 1, theoretical azimuth and range resolution are about 15.18 m and 7.38 m respectively. From eq.(10), the extent of the scene is about 7 km in azimuth direction and 2.5 km in range direction.


Fig 3 target scene near the equator


Fig 4 focused large scene using classical RMA

Table 2 performances list of 3 point targets

| P T | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| Ran. res. (m) | 7.33 | 7.34 | 7.35 |
| Azi. res. (m) | 15.20 | 15.22 | 15.18 |
| Ran. PSLR (dB) | -13.22 | -13.23 | -13.20 |
| Azi. PSLR (dB) | -13.25 | -13.22 | -13.19 |

Fig 5 focused large scene using modified RMA

## 4.1 simulation results of classical RMA

Fig 4 shows the SAR image focused by classical RMA. It can be concluded from the contour images in Fig 4 b that the edge point targets (1 and 3) are much more defocused than the central point target (2). In other words, the classical RMA can not focus the echo data from a large scene area in GEO SAR and should be modified.

## 4.2 simulation results of modified RMA

The focused scene using modified RMA is shown in Fig 5 including the contour images of 3 point targets. Table 2 gives the focusing performances of 3 point targets distributed around the scene. From Fig 5 and Table 2, the whole scene has been successfully focused, which indicates that modified RMA can handle with the spacevariance of Doppler parameters.
The simulation results validate the modified RMA have the capability to focus large scene in GEO SAR.

## References

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