# ROLL INVARIANT TARGET DETECTION BASED ON POLSAR CLUTTER MODELS 

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## 1. INTRODUCTION

In this paper, a method is proposed for detecting Polarimetric Synthetic Aperture Radar (PolSAR) targets. The proposed method is a combination of the Target Scattering Vector Model (TSVM) and the Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) detector. The TSVM provides an unique and roll-invariant decomposition of the observed target vector by means of four independent parameters. The combination of those two methods will allow the detection of any oriented targets (trihedral, dihedral, dipole, helix, . . .).

This paper is organized as follows. The context of the study is first described. Then, the TSVM algorithm is exposed. Next, the proposed algorithm for a roll-invariant target detection is presented. Then, some detection results are shown on a real PolSAR data-set acquired by the RAMSES sensor at X-band.

## 2. ROLL-INVARIANT DECOMPOSITION

### 2.1. Problem formulation

Let $\mathbf{k}_{\text {dip }}$ and $\mathbf{k}_{\text {dih }}$ be respectively the steering vectors in the Pauli basis of two oriented dipole and dihedral. They are are respectively defined by:

$$
\mathbf{k}_{d i p}=\frac{1}{\sqrt{2}}\left[\begin{array}{c}
1  \tag{1}\\
\cos (2 \psi) \\
\sin (2 \psi)
\end{array}\right] \text { and } \mathbf{k}_{d i h}=\left[\begin{array}{c}
0 \\
\cos (2 \psi) \\
\sin (2 \psi)
\end{array}\right]
$$

where $\psi$ is the orientation of the maximum polarization with respect to the horizontal polarization [1].
Consequently, for a roll invariant-target dipole or dihedral detection, the tilt angle $\psi$ should be removed. In 1993, Krogager has proposed an algorithm to derive $\psi$ which uses the phase difference between right-right $\left(S_{R R}\right)$ and left-left $\left(S_{L L}\right)$ circular polarizations of the scattering matrix $S$ [2]. $S_{R R}$ and $S_{L L}$ are respectively defined by $S_{R R}=\left(S_{H H}-S_{V V}+2 j S_{H V}\right) / 2$ and $S_{L L}=\left(S_{V V}-S_{H H}+2 j S_{H V}\right) / 2$.

The expression of the orientation angle given by Korgager is $\psi_{\text {Krogager }}=\left[\operatorname{Arg}\left(S_{R R} S_{L L}^{*}+\pi\right)\right] / 4$. This estimated orientation angle $\psi_{\text {Krogager }}$ is valid under certain condition on the target. To overcome this problem, authors propose to apply the TSVM method which provides an unique and roll-invariant decomposition of any targets [1].

### 2.2. The Target Scattering Vector Model

The TSVM, proposed by Touzi in 2007, consists in the projection in the Pauli basis of the scattering matrix con-diagonalized by the Takagi method [1]. It leads:

$$
\overrightarrow{e_{T}} \mathbf{S V}=m e^{j \Phi_{s}}\left[\begin{array}{ccc}
1 & 0 & 0  \tag{2}\\
0 & \cos (2 \psi) & -\sin (2 \psi) \\
0 & \sin (2 \psi) & \cos (2 \psi)
\end{array}\right]\left[\begin{array}{c}
\cos \alpha_{s} \cos \left(2 \tau_{m}\right) \\
\sin \alpha_{s} e^{j \Phi_{\alpha_{s}}} \\
-j \cos \alpha_{s} \sin \left(2 \tau_{m}\right)
\end{array}\right]
$$

The rotation angle $\psi$ is used for the subtraction of the target orientation from the target vector. This step is named desying. $\tau_{m}$ is the target helicity, it characterizes the symmetry of the target. $m$ is the maximum amplitude return. $\alpha_{s}$ and $\Phi_{\alpha_{s}}$ are the symmetric scattering type magnitude and phase. They are derived from the coneigenvalues $\mu_{1}$ and $\mu_{2}$ of the scattering matrix Sby:

$$
\begin{equation*}
\tan \left(\alpha_{s}\right) e^{j \Phi_{\alpha_{s}}}=\frac{\mu_{1}-\mu_{2}}{\mu_{1}+\mu_{2}} . \tag{3}
\end{equation*}
$$

Because of the coneigenvalue phase ambiguity, Huynen's orientation angle $\psi$ should be re-evaluated. To remove this ambiguity, the following relation is applied to restrict the interval of $\psi$ to $[-\pi / 4, \pi / 4]$ :

$$
\begin{equation*}
\overrightarrow{e_{T}} \mathbf{S V}\left(\Phi_{s}, \psi, \tau_{m}, m, \alpha_{s}, \Phi_{\alpha_{s}}\right)=\overrightarrow{e_{T}} \mathbf{S V}\left(\Phi_{s}, \psi \pm \frac{\pi}{2},-\tau_{m}, m,-\alpha_{s}, \Phi_{\alpha_{s}}\right) . \tag{4}
\end{equation*}
$$

As the last term of (2) is independent of the target orientation angle, it yields that the four parameters $m, \alpha_{s}, \Phi_{\alpha_{s}}$ and $\tau_{m}$ are rollinvariant. In the following, the TSVM method is first applied on the original PolSAR data-set to provide a roll-invariant target vector. To compute the target orientation angle with the TSVM decomposition, the following relation is implemented [3] [4] [5]:

$$
\begin{equation*}
\psi=\frac{1}{2} \operatorname{Arctan}\left(\frac{2 \Re \mathrm{e}\left\{\left(S_{H H}^{*}+S_{V V}^{*}\right) S_{H V}\right\}}{\Re \mathrm{e}\left\{\left(S_{H H}^{*}+S_{V V}^{*}\right)\left(S_{H H}-S_{V V}\right)\right\}}\right) \tag{5}
\end{equation*}
$$

### 2.3. Comparison between $\psi$ and $\psi_{K r o g a g e r}$

According to the TSVM, the following relation between the orientation angle $\psi$ estimated by the TSVM method and $\psi_{\text {Krogager }}$ estimated with the phase difference between right-right and left-left circular polarizations can be proved:

$$
\begin{equation*}
\psi=\psi_{\text {Krogager }}-\frac{1}{4} \operatorname{Arctan}\left(\frac{\tan \left(\alpha_{s}\right) \sin \left(\Phi_{\alpha_{s}}\right)}{\tan \left(\alpha_{s}\right) \cos \left(\Phi_{\alpha_{s}}\right)+\sin \left(2 \tau_{m}\right)}\right)+\frac{1}{4} \operatorname{Arctan}\left(\frac{\tan \left(\alpha_{s}\right) \sin \left(\Phi_{\alpha_{s}}\right)}{\tan \left(\alpha_{s}\right) \cos \left(\Phi_{\alpha_{s}}\right)-\sin \left(2 \tau_{m}\right)}\right) \tag{6}
\end{equation*}
$$

Fig. 1 shows a comparison between the orientation angle $\psi$ estimated via the TSVM and $\psi_{\text {Krogager }}$ as a function of three roll-invariant TSVM parameters: $\tau_{m}, \Phi_{\alpha_{s}}$ and $\alpha_{s}$. Fig. 1(a) shows the evolution of $\psi$ and $\psi_{\text {Krogager }}$ with the helicity $\tau_{m}$ for $\alpha_{s}=\pi / 3$ and $\Phi_{\alpha_{s}}=\pi / 3$. Fig. 1(b) and Fig. 1(c) show respectively this relation as a function of the target scattering phase $\Phi_{\alpha_{s}}$ for $\alpha_{s}=\pi / 3$ and $\tau_{m}=\pi / 8$, and as a function of $\alpha_{s}$ for $\Phi_{\alpha_{s}}=\pi / 3$ and $\tau_{m}=\pi / 8$. For $\tau_{m}=0$, the target is symmetric. It leads that $\psi$ is equal to $\psi_{\text {Krogager }}$, as observed in black in Fig. 1(a). Moreover, for a null target scattering phase $\Phi_{\alpha_{s}}, \psi_{\text {Krogager }}$ and $\psi$ are equal. Similar observations can be done for $\alpha_{s}=0$ and $\alpha_{s}=\pi / 2$ as shown in Fig. 1(c).

For $\tau_{m}=0, \Phi_{\alpha_{s}}=0, \alpha_{s}=0$ or $\alpha_{s}=\pi / 2$, the orientation angle estimated by the phase difference between right-right and left-left circular polarizations is equal to this estimated by the TSVM. It leads that both tilt angles are equal for a wide class of targets including trihedral, dihedral, helix, dipole, quarter wave, .. For all other cases, the orientation angle $\psi_{\text {Krogager }}$ is biased, and $\psi$ should be used instead for a roll-invariant target characterization.

## 3. ROLL-INVARIANT TARGET DETECTION

The general principle of the proposed roll-invariant target detection algorithm can be divided into five steps. First, the orientation angle is computed and the "roll-invariant" target vectir is extracted. Then, the covariance matrix of the clutter is estimated. Next,


Fig. 1. Comparison between $\psi$ and $\psi_{\text {Krogager }}$ : (a) as a function of $\tau_{m}$ for $\alpha_{s}=\pi / 3$ and $\Phi_{\alpha_{s}}=\pi / 3$, (b) as a function of $\Phi_{\alpha_{s}}$ for $\alpha_{s}=\pi / 3$ and $\tau_{m}=\pi / 8$ and (c) as a function of $\alpha_{s}$ for $\Phi_{\alpha_{s}}=\pi / 3$ and $\tau_{m}=\pi / 8$
the similarity measure between the steering vector and the "roll-invariant" target vector is computed. The false alarm probability is fixed, and finally we conclude or not on the detction.

### 3.1. GLRT-LQ detector

The Generalized Likelihood Ratio Test - Linear Quadratic (GLRT-LQ) detector can be used to detect a particular target. Let p be a steering vector and $\mathbf{k}$ the observed signal. The GLRT-LQ between $\mathbf{p}$ and $\mathbf{k}$ is given by [6]:

$$
\begin{equation*}
\Lambda([M])=\frac{\left|\mathbf{p}^{H}[M]^{-1} \mathbf{k}\right|^{2}}{\left(\mathbf{p}^{H}[M]^{-1} \mathbf{p}\right)\left(\mathbf{k}^{H}[M]^{-1} \mathbf{k}\right)} \stackrel{H_{1}}{\stackrel{\rightharpoonup}{H_{0}}} \lambda \tag{7}
\end{equation*}
$$

where $[M]$ is the covariance matrix of the population under the null hypothesis $H_{0}$, i.e. the observed signal is only the clutter.
In general, the covariance matrix is unknown. One solution consists in estimating the covariance matrix $[M]$ by $[\hat{M}]_{F P}$, the fixed point covariance matrix estimator. It is the maximum likelihood estimator of the normalized covariance matrix under the deterministic texture in a Spherically Invariant Random Process. Its expression is given by the solution of the following recursive equation [7]:

$$
\begin{equation*}
[\hat{M}]_{F P}=f\left([\hat{M}]_{F P}\right)=\frac{p}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_{i} \mathbf{k}_{i}^{H}}{\mathbf{k}_{i}^{H}[\hat{M}]_{F P}^{-1} \mathbf{k}_{i}} \tag{8}
\end{equation*}
$$

Replacing $[M]$ by $[\hat{M}]_{F P}$ in (7) leads to an adaptive version of the GLRT-LQ detector.

### 3.2. Optimal KummerU Detector (OKUD)

For a Fisher distributed texture, it has been proved that the target scattering vector follows a KummerU PDF [8]. Based on this multivariate statistics, the Optimal KummerU Detector can be constructed, its expression is given by :

$$
\begin{equation*}
\frac{U\left(p+\mathcal{M} ; 1+p-\mathcal{L} ; \frac{\mathcal{L}}{\mathcal{M} m}(\mathbf{k}-\mathbf{p})^{H}[M]^{-1}(\mathbf{k}-\mathbf{p})\right)}{U\left(p+\mathcal{M} ; 1+p-\mathcal{L} ; \frac{\mathcal{L}}{\mathcal{M} m} \mathbf{k}^{H}[M]^{-1} \mathbf{k}\right)} \underset{H_{0}}{\stackrel{H_{1}}{\gtrless}} \lambda \tag{9}
\end{equation*}
$$

where $m, \mathcal{L}$ and $\mathcal{M}$ are the Fisher parameters. $m$ is a scale parameter, $\mathcal{L}$ and $\mathcal{M}$ are two shape parameters. $U(\cdot ; \cdot ; \cdot)$ is the confluent hypergeometric function of the second kind (KummerU).

For a KummerU distributed clutter, the exact ML estimator of the covariance matrix [ $\hat{M}_{M L}$ ] is given by [9]:

$$
\begin{equation*}
\left[\hat{M}_{M L}\right]=\frac{p+\mathcal{M}}{N}\left(\frac{\mathcal{L}}{\mathcal{M} m}\right) \sum_{i=1}^{N} \frac{U\left(p+1+\mathcal{M} ; 2+p-\mathcal{L} ; \frac{\mathcal{L}}{\mathcal{M} m} \mathbf{k}_{i}^{H}\left[\hat{M}_{M L}\right]^{-1} \mathbf{k}_{i}\right)}{U\left(p+\mathcal{M} ; 1+p-\mathcal{L} ; \frac{\mathcal{L}}{\mathcal{M} m} \mathbf{k}_{i}^{H}\left[\hat{M}_{M L}\right]^{-1} \mathbf{k}_{i}\right)} \mathbf{k}_{i} \mathbf{k}_{i}^{H} \tag{10}
\end{equation*}
$$

## 4. DETECTION RESULTS ON A RAMSES X-BAND DATA-SET

In this section, a real data-set acquired by the RAMSES sensor at X-band is analyzed. Fig. 2 shows a colored composition in the Pauli basis of the target vector. This data-set is composed by two particular targets: a dihedral (in green) and a narrow diplane (in red). Both GLRT-LQ Krogager (tilt angle estimated by $\psi_{\text {Krogager }}$ ) and GLRT-LQ TSVM detectors are applied

(a)

(b)

|  | dihedral |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | GLRT-LQ | $\psi$ | $\alpha_{s}$ | $\Phi_{\alpha_{s}}$ | $\tau_{m}$ |  |
|  | 0.912 | 0.761 |  |  |  |  |
| TSVM | 0.956 | 0.770 | -1.453 | 0.450 | -0.178 |  |
| Pure target |  |  | 1.571 | $\infty$ | 0 |  |
|  | narrow diplane |  |  |  |  |  |
|  | GLRT-LQ | $\psi$ | $\alpha_{s}$ | $\Phi_{\alpha_{s}}$ | $\tau_{m}$ |  |
| Krogager | 0.828 | -0.023 |  |  |  |  |
| TSVM | 0.849 | -0.026 | 1.210 | -0.172 | 0.052 |  |
| Pure target |  |  | 1.249 | 0 | 0 |  |

(c)

Fig. 2. Toulouse, RAMSES PolSAR data, X-band ( $150 \times 150$ pixels). Colored composition in the Pauli basis of the target vector $[k]_{1}-[k]_{3}-[k]_{2}$. Images containing a dihedral (a) and a narrow diplane (b). (c) Detector characteristics for the dihedral and the the narrow diplane
on this data-set. Fig. 2(c) shows the criterion characteristics for the dihedral and narrow diplane. As those two targets have theoretically a null helicity $\tau_{m}$, both detectors should have similar performance. For a fixed false alarm probability of $5 \times 10^{-3}$, the detection treshold is $\lambda=0.931$. For the dihedral, The GLRT-LQ TSVM is able to detect the target $(0.956>\lambda)$ whereas the GLRT-LQ Krogager detector fails $(0.912<\lambda)$.

## 5. CONCLUSION

In this paper, authors have proposed to the use target scattering vector model to extract the roll-invariant target vector. Some comparisons have been done between the orientation angle estimated with the phase difference between right-right and leftleft circular polarizations and this issued from the TSVM. Next, authors have proposed to use the TSVM for a roll-invariant target detection. The GLRT-LQ similarity measure has been implemented and validated on high resolution PolSAR data for the detection of particular targets. In the final version of this paper, authors propose to use Optimal KummerU Detector as similarity measure. It takes into account the statistics of the PolSAR clutter. Detection performance of this new detector will be analyzed.

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