# 3D VELOCITY MODEL AND RAY TRACING OF ANTENNA ARRAY GPR 

Xuan Feng ${ }^{I}$, Wenjin Liang ${ }^{I}$, Qi Lu ${ }^{I}$, Cai Liu ${ }^{I}$, Lili Li ${ }^{l}$, Lilong Zou ${ }^{I}$, and Motoyuki Sato ${ }^{2}$<br>${ }^{1}$ College of Geo-Exploration Science and Technology, Jilin University, Changchun 130026, China<br>${ }^{2}$ Center for Northeast Asian Studies, Tohoku University, 41 Kawauchi, Aob-ku, Sendai 980-8576, Japan

## 1. INTRODUCTION

It is often advantageous to acquire ground- penetrating radar (GPR) measurements from antennas that are offset from the air-ground interface by a nonnegligible distance, either because the ground surface is rough, or because measurements must be collected remotely [4], for example, the detection of buried landmines. In this case, the GPR antenna(s) must be elevated above the ground surface [1]. The migration technique [3] is now commonly used to process GPR data, and has been in use for almost five decades in seismic reflection surveys. Generally the diffraction stacking migration or Kirchhoff migration need compute the travel time. When the height of ground surface varies largely in the very rough ground area, for example mound, the travel time surface will be affected by the ground surface. In the case we need to calculate precise travel time.

Ray tracing is the technique that can offer precise both the travel time and ray path of electromagnetic wave in rough ground area. To develop the ray tracing technique, we need the 3 D velocity model of the measurement area.

## 2. ANTENNA ARRAY GPR SYSTEM

Based on the transmit antenna, receive antenna, scanning platform, and vector network analyzer (VNA), we constructed a stepped-frequency (SF) GPR system depicted in Fig. 1 [5]. Three pairs of Vivaldi antennas were used to configure a symmetrical CMP antenna array. In each pair of antennas, one is used to transmit the signal while the other is used to receive it [6]. M. Sato [2] described the characteristics of the Vivaldi antenna designed by ourselves.

## 3. ESTIMATING 3D VELOCITY MODEL

To acquire the information of the 3D velocity model, we have to estimate the both ground surface topography and velocities. Because the propagation velocity of electromagnetic wave in air is known, we need to estimate the soil velocity.

### 3.1. Estimation of Ground Surface Topography



Fig. 1. The SF GPR measurement system setup. (a) scene. (b) Disassemble Vivaldi antenna array. In most cases, by far the brightest reflector visible in a GPR images is the air-ground surface itself. This naturally gives rise to a method, searching for the brightest pixel in the GPR profile, by which the surface profile may be estimated. But in the sharp variable surface case, the method can not accurately estimate the surface topography, because of the diffraction waves.

To improve the accuracy of surface topography estimation, we propose that migration technique is used to process GPR data before searching the brightest pixel in the GPR profile. The Kirchhoff migration was used to move reflectors into their true positions and collapses diffractions [7]. After migration, we can achieve the accurate surface topography through searching for the brightest pixel in the GPR profile [5].

### 3.2. Estimation of Soil Velocity

Velocity can be evaluated from the CMP data gather acquired by our GPR system. By quantifying the similarities among signals in the CMP data gather, it is then possible to evaluate the root-mean-square (RMS) velocity. The similarity is usually evaluated by analyzing the coherence panels, which is called velocity spectrum in seismic data processing. The coherence, calculated in the plane, is a maximum for the correct velocity. To interpret the RMS velocity derived from the multi-offset data, it is necessary to calculate interval velocity that corresponds to the dielectric constant of medium by the Dix formula [6].

## 4. RAY TRACING

The basic idea is to find the first-arrival traveltimes by using Fermat's principles in a velocity model. According to Fermat's principle, the path with the smallest traveltime is the one best approximating the ray trajectory. It is possible to determine the traveltime between two arbitrary points. Fig. 2 shows a 2D velocity model. The velocity is C in the air layer and V in the subsurface layer. B is the arbitrary subsurface scattering point and A is the arbitrary transmitter or receiver. The ray path between A and B is ARiB. Ri is the arbitrary point in the ground


Fig. 2. sketch figure of ray path.


Fig. 3. Metal ball.

(b) top view

(a) side view

Fig. 4. The ray path in the mound case.
surface. So if the distance between A and Ri is dil and the distance between Ri and B is di2, the travel time between $A$ and $B$ is:
$T_{i}=\frac{d_{i 1}}{C}+\frac{d_{i 2}}{V}, \mathrm{i}=1, \ldots, \mathrm{n}$.
Then we can define the $\mathrm{T}_{\mathrm{j}}$ in the ground surface, depending on the Fermat's principle, $T_{j}=\min \left(T_{i}\right)$

So, the ray path between $A$ and $B$ is $A R_{j} B$ and the first-arrival travel time is $\mathrm{T}_{\mathrm{j}}$. Using the approach, we calculate the ray path.


Fig. 5. A profile through metal ball buried in the mound and travel time trajectory.

## 5. EXPERIMENT

We buried a small metal ball whose radius is about 6 cm in the homogenous soil and the ground surface is mound, shown in Fig. 3, in laboratory. Then we measured it in C-scan model using GPR. The operational frequency
range is from 300 MHz to 6 GHz . The number of frequency points is 401 . The distance between two antennas is 6 cm . The height of antenna is 8 cm and the depth of metal ball is 10 cm . The x interval and y interval are 1 cm . Using the ray tracing, we calculate the ray path for the experiment data. Fig. 4 shows the ray path among one subsurface scattering point and all antenna position in C-scan model. From the figure, we can find the ray path distributed irregularly. Fig. 5 shows one vertical profile of the survey line above the metal ball and the travel time trajectory. From the figure, we can find the travel time trajectory and the diffraction signal from the metal ball correspond each other very well.

## 6. CONCLUSIONS

Depending on the midpoint symmetrical antenna array configuration, the GPR system can estimate the 3D velocity model including ground surface topography and velocity. The soil velocity can be estimated by the velocity spectrum technology and Dix equation. The ground surface topography can be searched precisely after migration processing that can focus the scattered wave and reconstruct the image of the ground surface. The prior information of 3D velocity model is possibly used for ray tracing. The technique of ray tracing can offer precise ray path and travel time trajectory of electromagnetic wave in rough ground area.

## 7. ACKNOWLEDGEMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 40704020, 973 Program under Grant 2009CB219301, "211 Project" of Jilin University, and "985 platform" of Jilin University.

## 8. REFERENCES

[1] B. Sai and L. P. Ligthart, "GPR phase-based techniques for profiling rough surfaces and detecting small, lowcontrast landmines under flat ground," IEEE Transactions on Geoscience and Remote Sensing, vol. 42, pp. 318 -326, Feb. 2004.
[2] M. Sato, Y. Hamada, X. Feng, F.-N. Kong, Z. Zeng, and G Fang, "GPR using an array antenna for landmine detection," Near Surface Geophysics, Vol. 2, pp. 7-13, Feb. 2004.
[3] Ö. Yilmaz, Seismic Data Analysis, Tulsa, OK: Society of Exploration Geophysicists, 2001.
[4] P. D. Walker and M. R. Bell, "Noniterative techniques for GPR imaging through a nonplanar air-ground interface," IEEE Transactions on Geoscience and Remote Sensing, vol. 40, pp. 2213-2223, Oct. 2002.
[5] X. Feng, M. Sato, C. Liu, and Y. Zhang, "Profiling the Rough Surface by Migration," IEEE Geoscience and Remote Sensing Letters, vol.6, no.2, pp. 258-262, April 2009.
[6] X. Feng, M. Sato, Y. Zhang, C. Liu, F.S. Shi, and, Y.H. Zhao, "CMP Antenna Array GPR and Signal-toClutterRatio Improvement," IEEE Geoscience and Remote Sensing Letters, vol.6, no.1, pp. 23-27, January 2009.
[7] X. Feng and M. Sato, "Pre-stack migration applied to GPR for landmine detection," Inverse Probl., vol. 20, no. 6, pp. S99-S115, Dec. 2004.

