Environmental Geodesy

Lecture 5 (February (15)22, 2011): Earth's Rotation
- Introductory Remarks: Main Characteristics
- Theory
- Polar Motion
- Length of Day

Introductory Remarks

Normally separate discussion of orientation and rotational speed

Variations in orientation:

- Precession and forced nutation: changes in the position of the axis of rotation in space
- Wobble: changes of the axis of rotation with respect to the Earth's crust
- polar drift: secular change in position of the rotation axis relative to Earth's crust

Cause of precession and forced nutation:

Pull of Moon, Sun and planets on the non-spherical, rotating Earth with an equatorial plane tilted against the ecliptic plane and the plane of the Moon's orbit.

Cause of wobble:

Internal Earth system processes

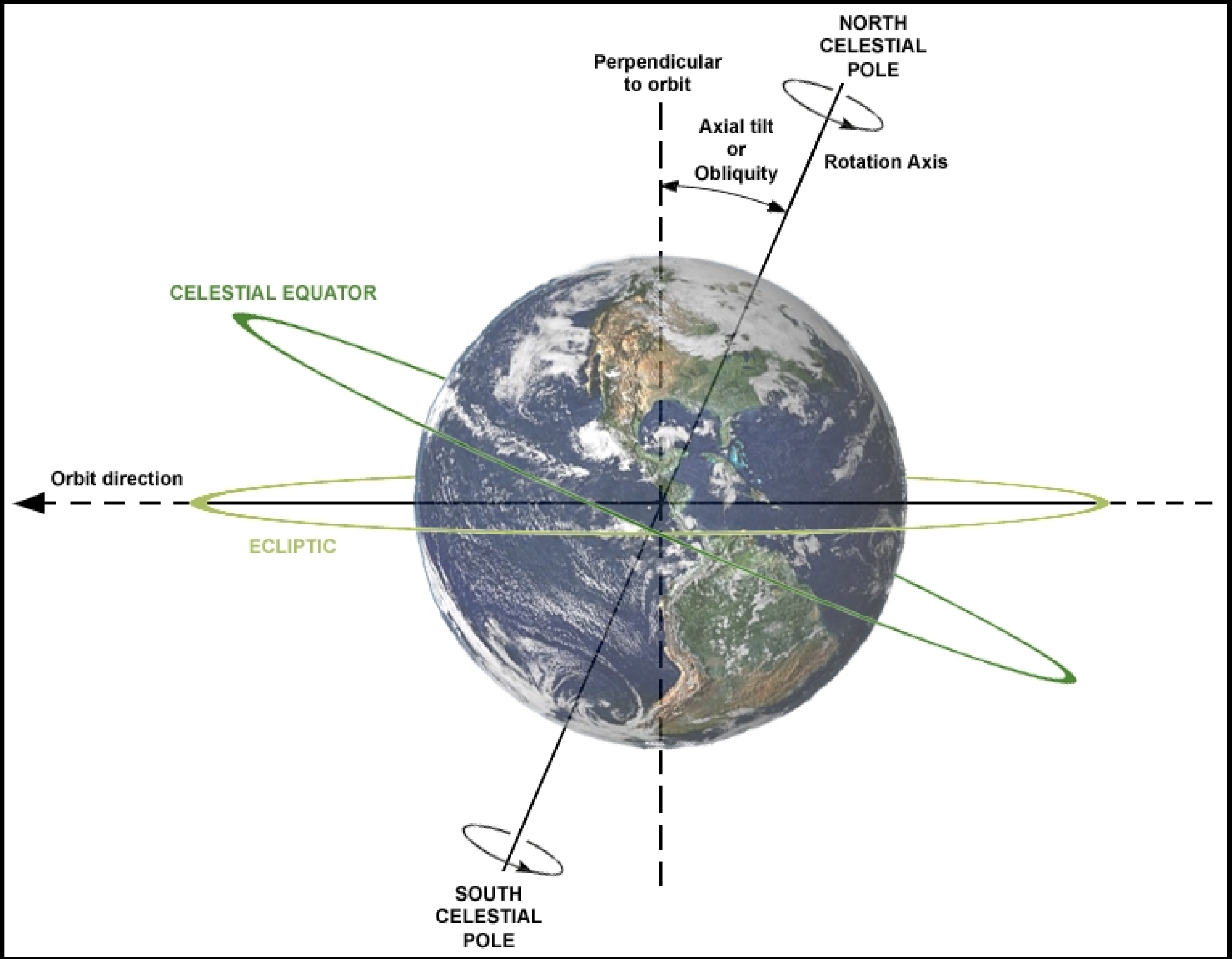
Cause of polar drift:

Mass redistribution in the water cycle; redistribution of continents;

Variations in LOD:

Both external (tidal forces) and internal processes

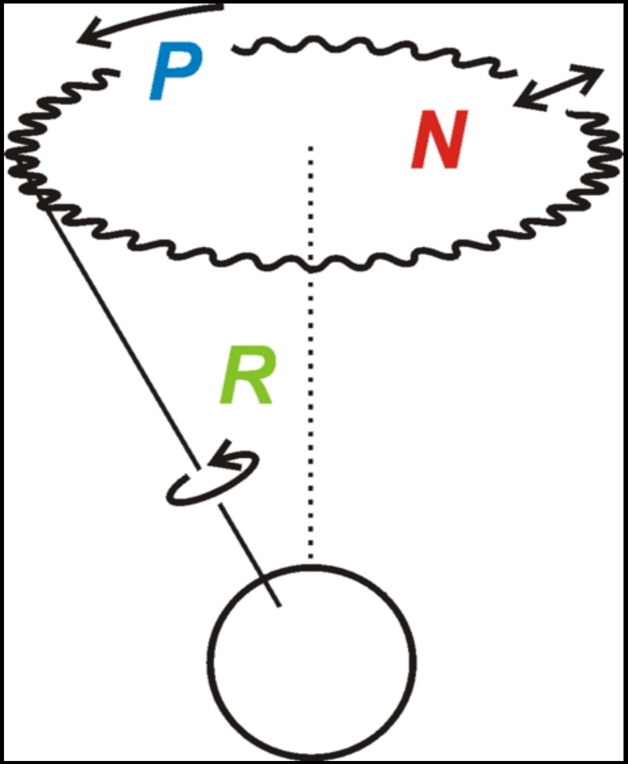
Introductory Remarks



Precession:

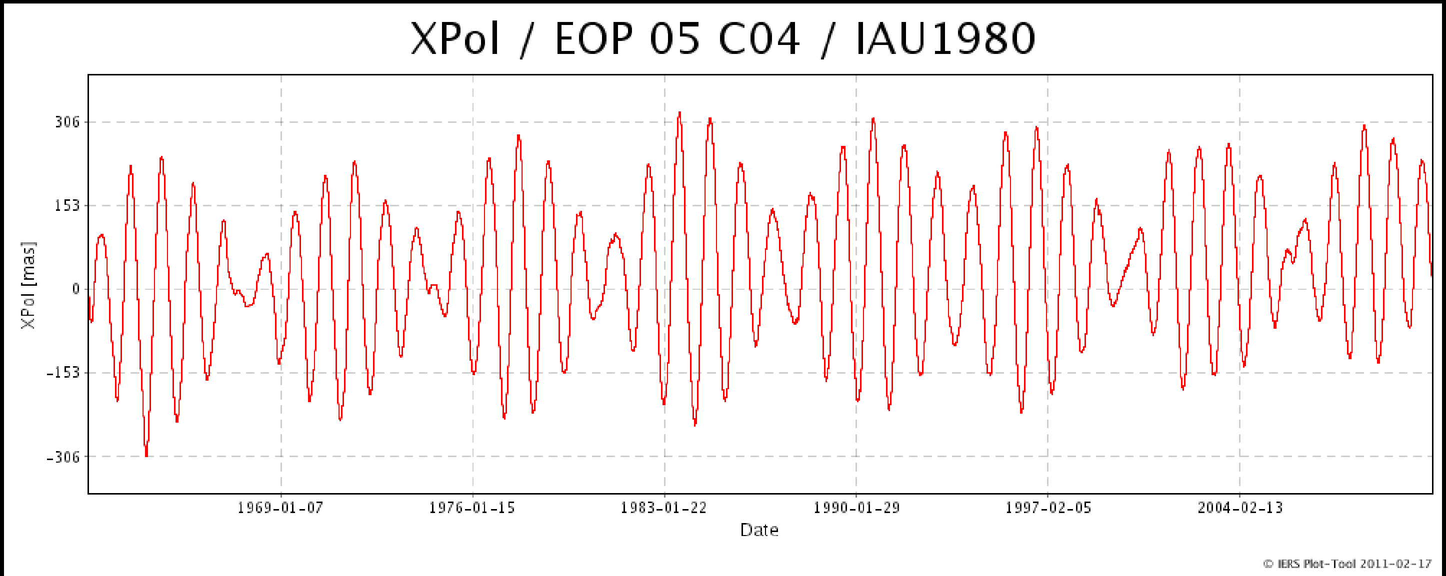
- largest secular motion of the rotation axis in space;
- Celestial pole moves (westward) around the pole of the ecliptic with a period of ~26,000 years (~20 minutes per year) plus other periods;
- obliquity remains within 23.5±1.3 degrees (period 41,000 years);
- eccentricity varies with 100,000 and 413,000 years.

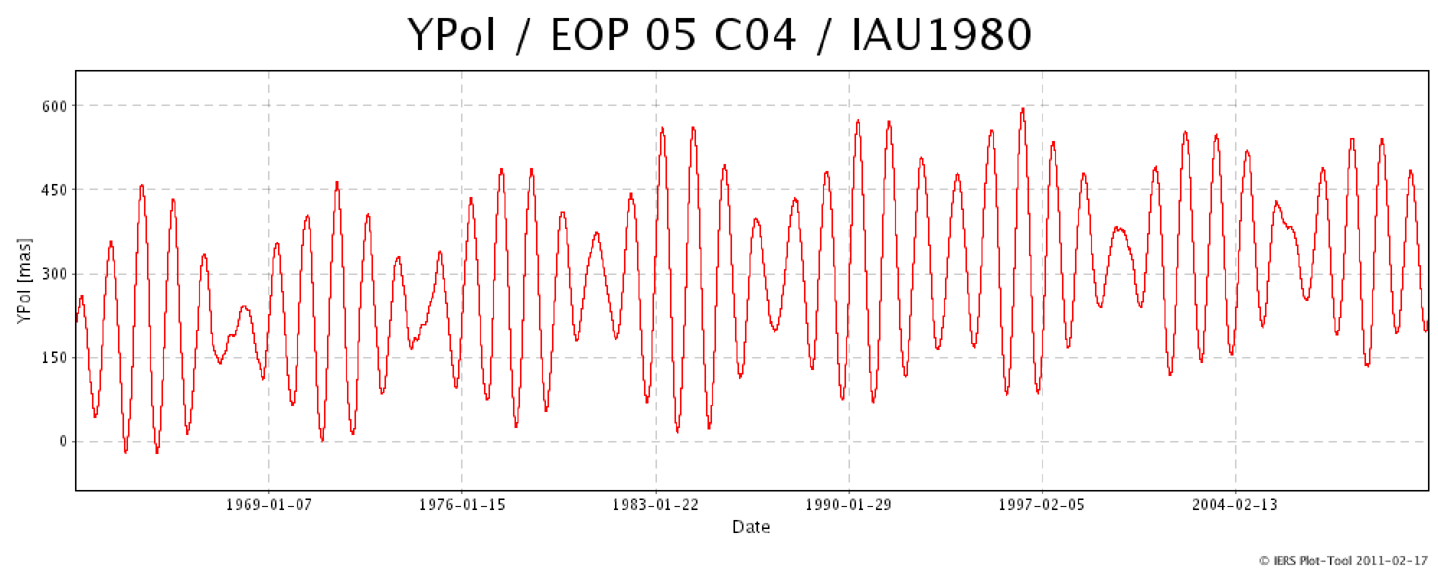
Introductory Remarks



Forced nutation:
Periodic variations (small tipping of the rotation axis) of up to 18.6 years caused by tidal forces.

Introductory Remarks





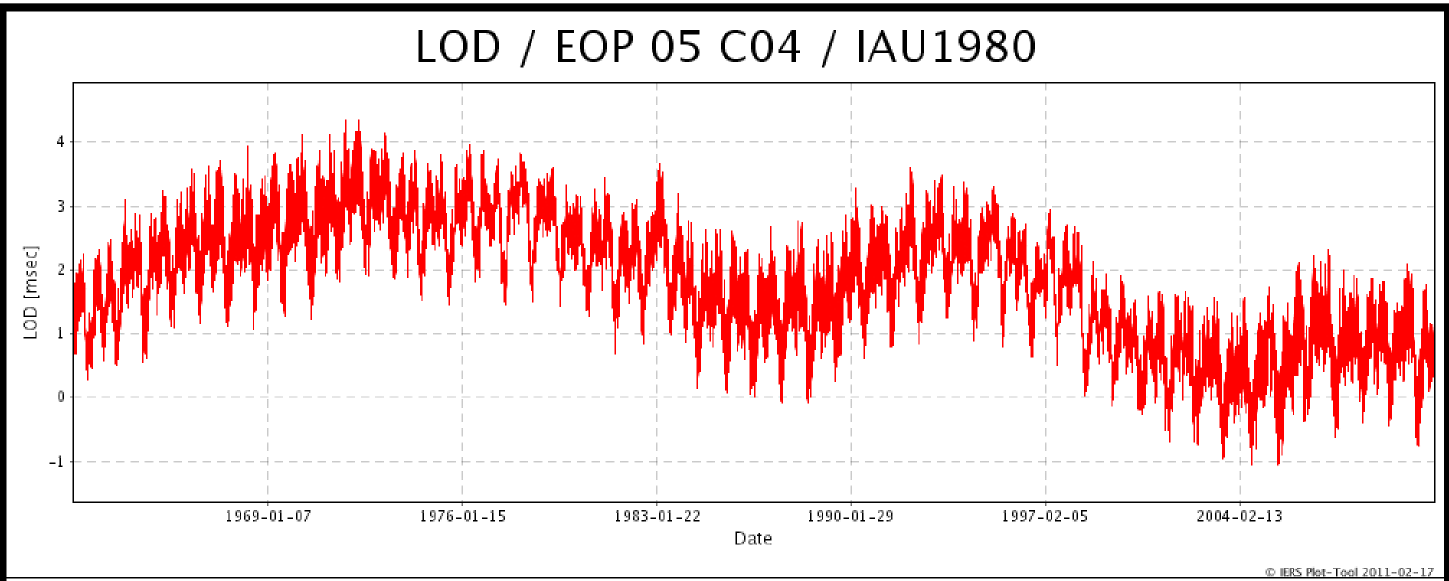
Observed polar motion:
1 mas = ~31 mm
Range on the order of 20 m

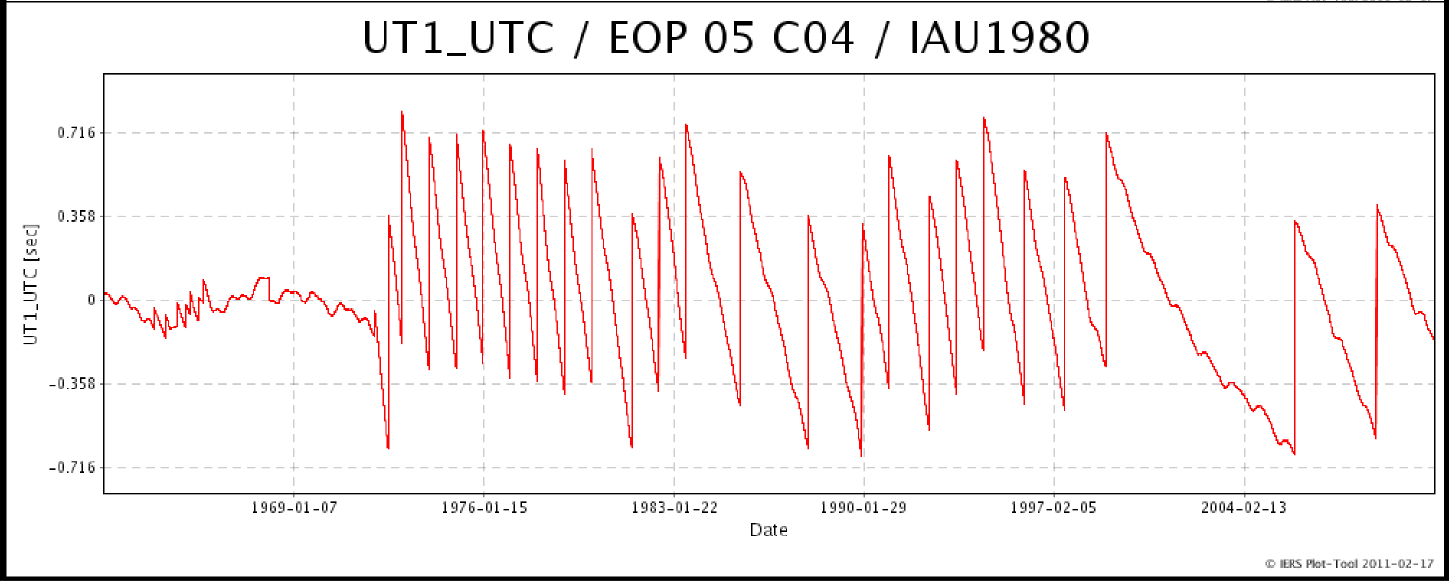
Introductory Remarks

Wobbles:
- changes in the figure axis of the solid Earth with respect to the rotation axis;
- first rotational eigenmode with a period close to 433 days; Chandler wobble; damped wobble needs to be maintained by excitation;
- annual wobble; forced by seasonal variations;
- "nearly diurnal free wobble" and "free core nutation"; second rotational eigenmode caused by the fluid core; period depends on ellipticity of CMB; corresponds to a nutation of ~460 days.
- "free inner core nutation"; third rotational eigenmode, due to the interaction of inner and outer core.

Polar drift:
- main contribution from mass relocation during ice ages and postglacial rebound.

Introductory Remarks





LOD: Deviation of length of day from 86,400 s.

UT1: mean solar time

UTC: Coordinated Universal Time (derived from TAI: International Atomic Time)

Introductory Remarks

Length of day changes:

- coupled to orientation changes through changes in the moment of inertia;
- short-period: tides, weather
- seasonal: water cycle, seasonal mass relocation
- decadal: core-mantel, climate
- secular: tidal acceleration/friction

Introductory Remarks

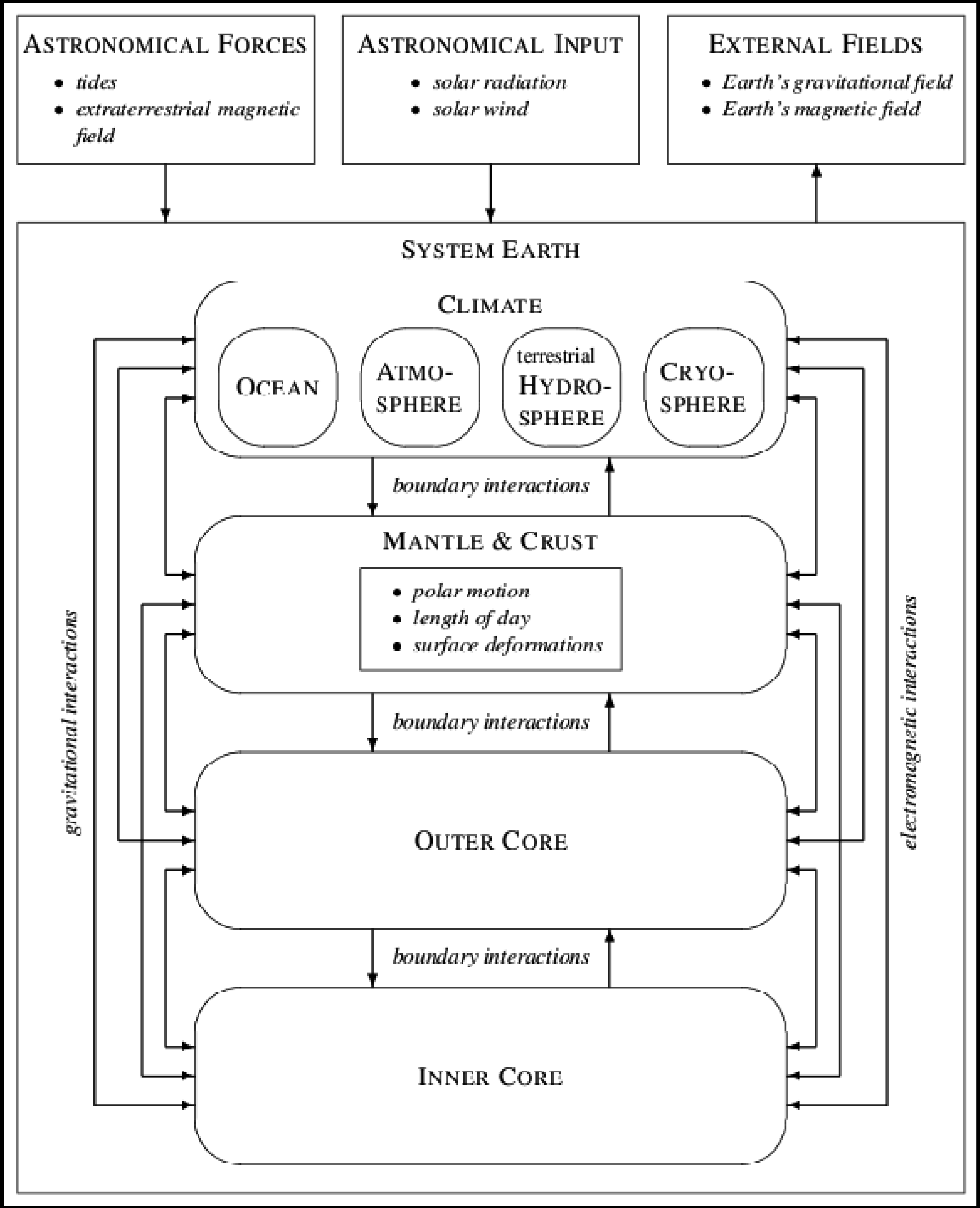
Precession:

- well understood, models with high accuracy in prediction available;
- will not be consider further.

Forced Nutation:

- of interest for studies of the rotational response of the Earth to very well known torque due to Sun, Moon and Planets;
- response is dominated by the presence of the outer (fluid) and inner (solid) cores, viscoelastic properties of the mantle and cores, and the presence of the oceans
- increase in accuracy requires transition for rigid earth models to elastic models;

Theory



The Earth system:

A system composed of physically defined subsystems.

Modules interact through boundary conditions and far-field interactions

Couplings between different subsystems defined as meaningful physical quantities **independent of structure and dynamics of the subsystems**

External excitations may be acting on several subsystems (such as tidal potential, magnetic field)

Theory

Two principle approaches:
(1) use momentum balance, all volume and surface forces, and external torque to model the rotation of each module;
(2) use the angular momentum balance for each module to compute its rotation.

(2) has been used widely for analytical approaches;

(1) is increasingly used for numerical system models.

Theory

The angular momentum balance is given by

$$\frac{d}{dt}\mathbf{H} = \mathbf{L}, \qquad (1)$$

with $\mathbf{H}$ the total angular momentum of the body, and $\mathbf{L}$ the external torque in an inertial frame of reference. Transforming this balance into a reference frame rotating with $\boldsymbol{\Omega}(t)$ results in

$$\mathbf{H} = \boldsymbol{\Theta} \cdot \boldsymbol{\Omega} + \mathbf{h}, \qquad (2)$$

where $\mathbf{h}$ is the relative angular momentum of the body, and $\boldsymbol{\Theta}$ is the (time-dependent) inertia tensor defined as

$$\boldsymbol{\Theta}(t) = \int_{V(t)} \rho(\mathbf{x}, t)(\mathbf{x}^2 \boldsymbol{I} - \mathbf{x} \otimes \mathbf{x})dV \qquad (3)$$

($\mathbf{x} \otimes \mathbf{x}$ results in a tensor with components $c_{ij} = x_i x_j$). In a reference frame rotating with angular velocity $\boldsymbol{\Omega}$ relative to the inertial frame, the angular momentum balance is written as

$$\boldsymbol{\Omega} \times (\boldsymbol{\Theta} \cdot \boldsymbol{\Omega} + \mathbf{h}) + \dot{\mathbf{h}} + \frac{d}{dt}(\boldsymbol{\Theta} \cdot \boldsymbol{\Omega}) = \mathbf{L}. \qquad (4)$$

This equation is non-linear in $\boldsymbol{\Omega}$ and describes the global rotation of an arbitrary body. In general, the external torque $\mathbf{L}$, the total angular momentum $\mathbf{H}$, the relative angular momentum $\mathbf{h}$ and the inertia tensor $\boldsymbol{\Theta}$ are all time-dependent parameters.

Theory

- Issue is the choice of Ω.
- Many earlier studies use Ω = const.
- Recent studies use a body-fixed system, although there are difference of how to fix the system to the rotating Earth

For a nearly constant angular speed, a perturbation approach with

$$\boldsymbol{\Omega}(t) = \Omega_0(\mathbf{e}_z + \mathbf{m}(t)) = (m_1(t), m_2(t), 1 + m_3(t))^t \Omega_0 \tag{5}$$

$$\mathbf{H}(t) = \boldsymbol{\Theta}(t) \cdot \boldsymbol{\Omega}(t) + \mathbf{h}(t) \tag{6}$$

$$\boldsymbol{\Theta}(t) = \boldsymbol{\Theta}_0 + \mathbf{c}(t), \tag{7}$$

where m_i, h_i and c_{ij} are small in first order, can be used to rewrite the system of equations (4). The coordinate axes are aligned with the main axes of the moment of inertia tensor, i.e., this tensor is given as

$$\boldsymbol{\Theta}_0 = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}. \tag{8}$$

Title, 4

Theory

Dropping all terms of order higher than one in system (4) leads to the Euler-Liouville-equations (ELE)

$$\boldsymbol{\Omega}_0 \times \mathbf{h} + \Omega_0 \boldsymbol{\Omega}_0 \times (\boldsymbol{\Theta}_0 \cdot \mathbf{m}) + \Omega_0 \mathbf{m} \times (\boldsymbol{\Theta}_0 \cdot \boldsymbol{\Omega}_0) + \boldsymbol{\Omega}_0 \times (\boldsymbol{c} \cdot \boldsymbol{\Omega}_0) + \dot{\mathbf{h}} + \Omega_0 \boldsymbol{\Theta}_0 \cdot \dot{\mathbf{m}} + \dot{\boldsymbol{c}} \cdot \boldsymbol{\Omega}_0 = \mathbf{L}, \tag{9}$$

where the dotted quantities indicate their derivatives with respect to time. Assuming rotational symmetry (i.e., $A = B$), introducing the complex quantities

$$\begin{aligned} m &= m_1 + i m_2 \\ c &= c_{13} + i c_{23} \\ h &= h_1 + i h_2 \\ L &= L_1 + i L_2 \end{aligned} \tag{10}$$

and defining the excitation functions

$$\Psi^{\mathrm{PM}} = \frac{-1}{\Omega_0^2 (C - A)} (\Omega_0 \dot{c} + i \Omega_0^2 c + \dot{h} + i \Omega_0 h - L) \tag{11}$$

$$\Psi^{\mathrm{LOD}} = \frac{-1}{\Omega_0 C} (\Omega_0 c_{33} + h_3 - L_3), \tag{12}$$

Title, Text,

results in the final form of the linear equations for PM and length-of-day (LOD) changes, i.e.

$$\frac{\dot{m}}{\sigma_r} - im = \Psi^{\mathrm{PM}} \tag{13}$$

$$\dot{m}_3 = \dot{\Psi}^{\mathrm{LOD}}, \tag{14}$$

where the wobble frequency σ_r is given by

$$\sigma_r = \Omega \frac{(C-A)}{A}. \tag{15}$$

For a rigid body with $\dot{\Theta} = 0$ and in a body-fixed reference frame (i.e. $\mathbf{h} = 0$), the excitation functions (11) and (12) in the absence of external torque $\mathbf{L}$ reduce to

$$\Psi^{\mathrm{PM}} = \frac{-1}{\Omega_0^2 (C-A)} (\Omega_0 \dot{c} + i\Omega_0^2 c) \tag{16}$$

$$\Psi^{\mathrm{LOD}} = \frac{-1}{C} (c_{33}). \tag{17}$$

The ELE given in (13) and (14) show that in the linearized case PM and

Theory

results in the final form of the linear equations for PM and length-of-day (LOD) changes, i.e.

$$\frac{\dot{m}}{\sigma_r} - im = \Psi^{\mathrm{PM}} \tag{13}$$

$$\dot{m}_3 = \dot{\Psi}^{\mathrm{LOD}}, \tag{14}$$

where the wobble frequency σ_r is given by

$$\sigma_r = \Omega \frac{(C - A)}{A}. \tag{15}$$

The ELE given in (13) and (14) show that in the linearized case PM and LOD changes are decoupled. For polar motion, the equations describe a linear oscillator, while LOD changes can be computed directly by integration of the respective excitation function over time.

Theory

For a rigid body with $\dot{\boldsymbol{\Theta}} = 0$ and in a body-fixed reference frame (i.e. $\mathbf{h} = 0$), the excitation functions (11) and (12) in the absence of external torque $\mathbf{L}$ reduce to

$$\Psi^{\mathrm{PM}} = \frac{-1}{\Omega_0^2 (C - A)} (\Omega_0 \dot{c} + i \Omega_0^2 c) \tag{16}$$

$$\Psi^{\mathrm{LOD}} = \frac{-1}{C} (c_{33}). \tag{17}$$

Theory

Period of the eigenmode:

- Euler predicted 305 days;
- misled the search for the eigenmode in latitude observations;
- Chandler (1891) found a mode at 437 days - Chandler Wobble;
- Newcomb explained the differences as due to deformation, ocean, ...
- elastic deformations change period from 305 to 445 days;
- ocean (pole tide) shorten the period;
- viscous response of mantle lengthen the period;
- viscous response of mantle dampen the wobble;
- presence of other layers (inner and out core) also affect the wobble period and give rise to other free modes;

Theory

Separating the angular momentum into an atmospheric part $\mathbf{H}_A$, and a solid Earth part $\mathbf{H}_E$ and doing the same for the excitation functions leads to

$$\mathbf{H} = \mathbf{H}_{\mathrm{E}} + \mathbf{H}_{\mathrm{A}} \tag{20}$$

$$\Psi^{\mathrm{PM}} = \Psi_{\mathrm{E}}^{\mathrm{PM}} + \Psi_{\mathrm{A}}^{\mathrm{PM}} \tag{21}$$

$$\Psi^{\mathrm{LOD}} = \Psi_{\mathrm{E}}^{\mathrm{LOD}} + \Psi_{\mathrm{A}}^{\mathrm{LOD}}. \tag{22}$$

Barnes *et al.* (1983) derive approximations for Ψ_{A}, which express these excitation functions in terms of the wind and surface pressure fields. However, they combine the angular momentum change of the atmosphere and the deformations of the Earth due to atmospheric loading to "Atmospheric Angular Momentum Functions (AAMF)", which is not desirable since the latter are depending on the Earth model while the first is not.

Angular momentum exchange between atmosphere and ocean is considered to be small.
Therefore, assumption often is $\mathbf{H}_E = -\mathbf{H}_A$.

Theory

Strictly speaking, the ELE describe the rotation of the Earth without a core. Is, in general, the body separable into layers which may rotate with respect to each other, then for each layer the angular momentum balance has to be considered, i.e.

$$\boldsymbol{\Omega} \times \sum_{i=1}^{n} \mathbf{H}^i + \frac{d}{dt} \sum_{i=1}^{n} \mathbf{H}^i = \mathbf{L} \tag{18}$$

$$\boldsymbol{\Omega} \times \mathbf{H}^i + \frac{d}{dt} \mathbf{H}^i = \mathbf{K}^i \ , \ i = 1, \ldots, n-1$$

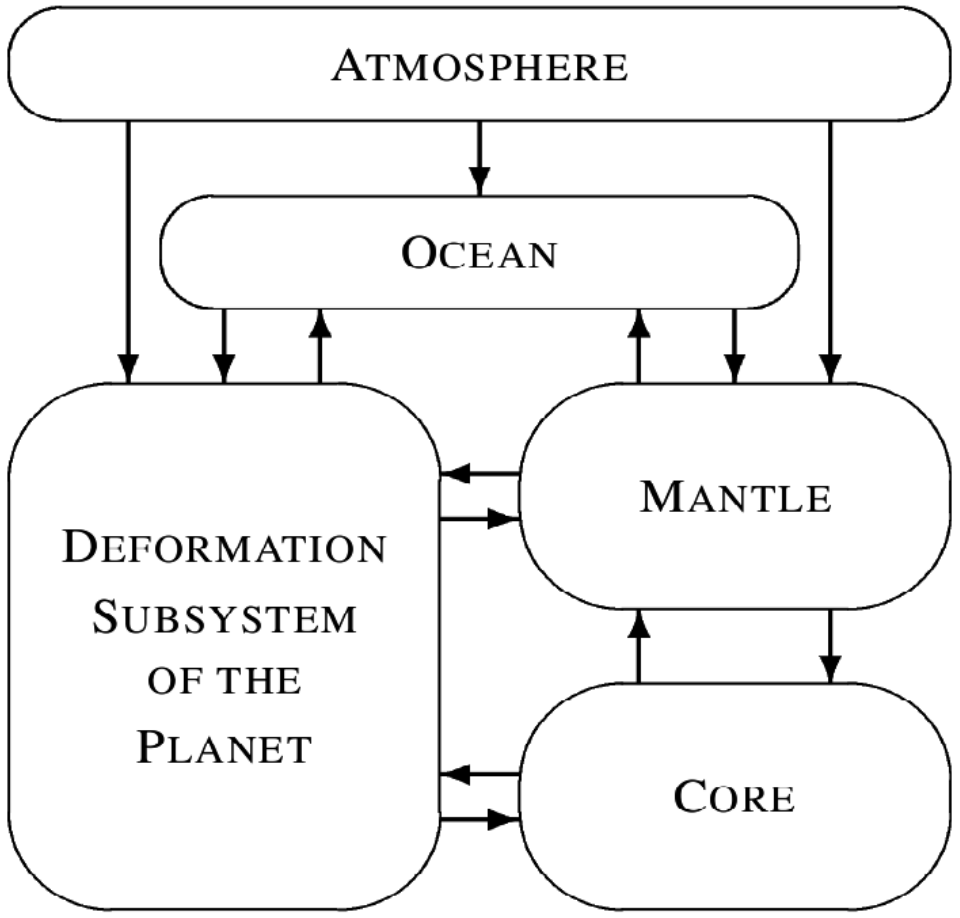
where $\mathbf{H}^i$ is the angular momentum of the i-th layer and $\mathbf{K}^i$ the torque acting on that layer, which depends on the coupling between the layers. It should be mentioned here, that for a simple Earth model with a mantle and a fluid core, the expression for σ_r has to be changed to

$$\sigma_r = \Omega_0 \frac{C_{\mathrm{M}} - A_{\mathrm{M}}}{A_{\mathrm{M}}}, \tag{19}$$

Theory

A simple, modular Earth system model

Dynamical Integrate Modular Earth Rotation System (DIMERS)



Modules of DIMERS:

(i) a quasi rigidly rotating core,

(ii) a quasi rigidly rotating mantle including crust,

(iii) a non-global equilibrium ocean,

(iv) an autonomous atmosphere,

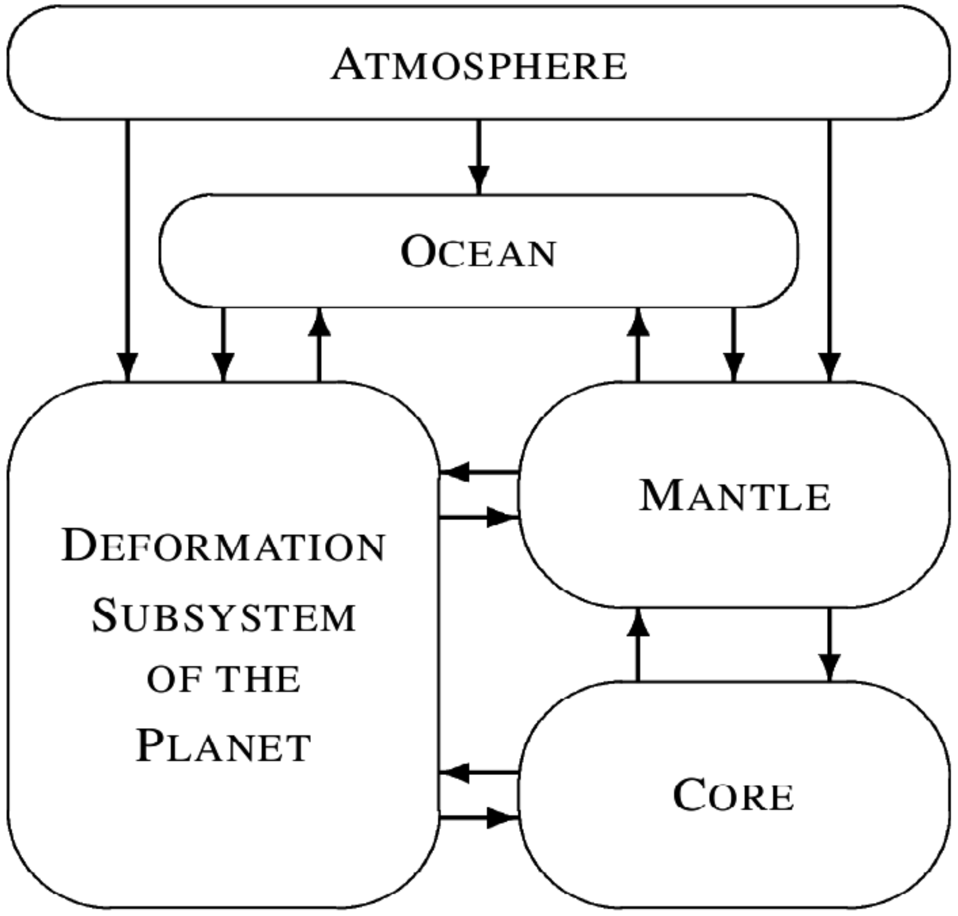
(v) a deformation system which represents the deformation of core, mantle and crust disregarded in (i) and (ii)

Coordinate system for all subsystem: dynamical Tisserand system of the mantle (DTM) with origin placed in the CoM of the mantle

Theory

A simple, modular Earth system model

Dynamical Integrate Modular Earth Rotation System (DIMERS)



Internals of the Subsystems:

Atmosphere: represented through prescribed 'observations'.

Ocean: non-global equilibrium ocean which reacts on

- variation of rotational potential and the secondary gravitational potential due to deformation of the planet
- air pressure variations and sea floor elevations

Mantle: linearised Euler-Liouville equation with damping with Q_m for the mantle

Core: Poincaré flow, i.e. rigid rotation relative to the mantle-fixed system. Damping with Q_c (core).

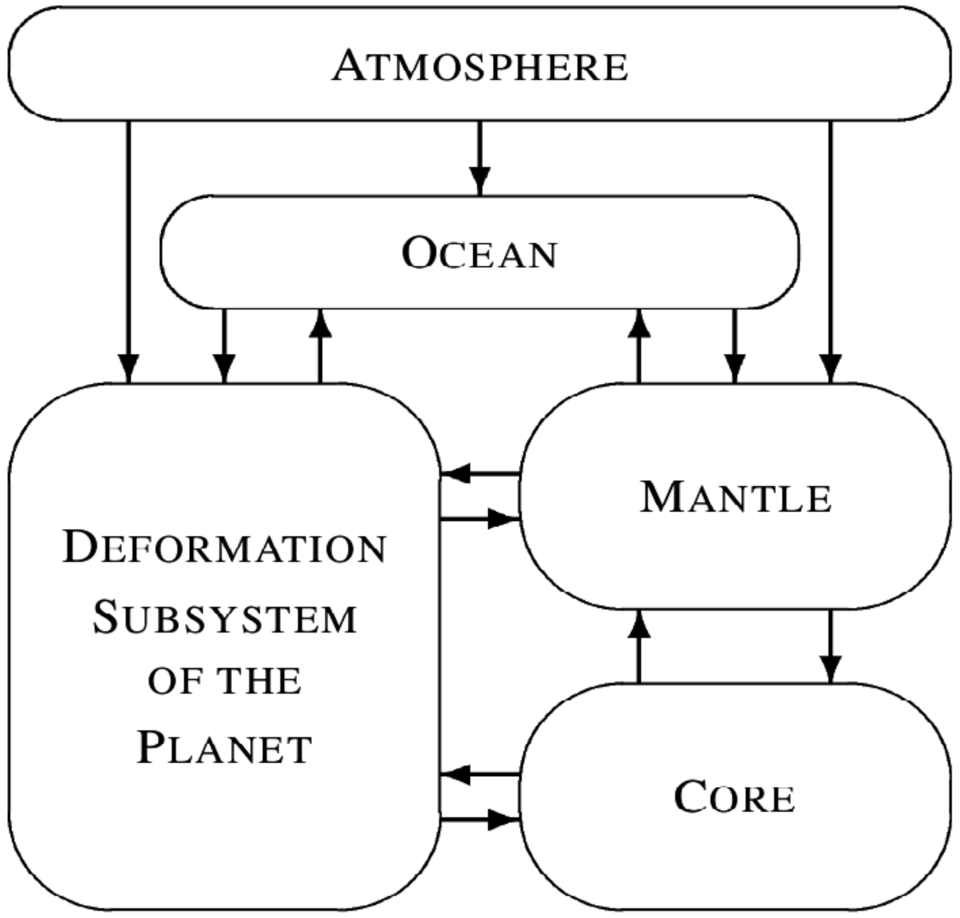
Deformation:

- provides inertia tensor of mantle and core based on Love-Shida numbers for PREM
- provides surface displacements for boundaries

Theory

A simple, modular Earth system model

Dynamical Integrate Modular Earth Rotation System (DIMERS)



Interactions:

Atmosphere & deformation and ocean: air pressure

Atmosphere & mantle: angular momentum exchange (torque)

Ocean & deformation: ocean bottom pressure

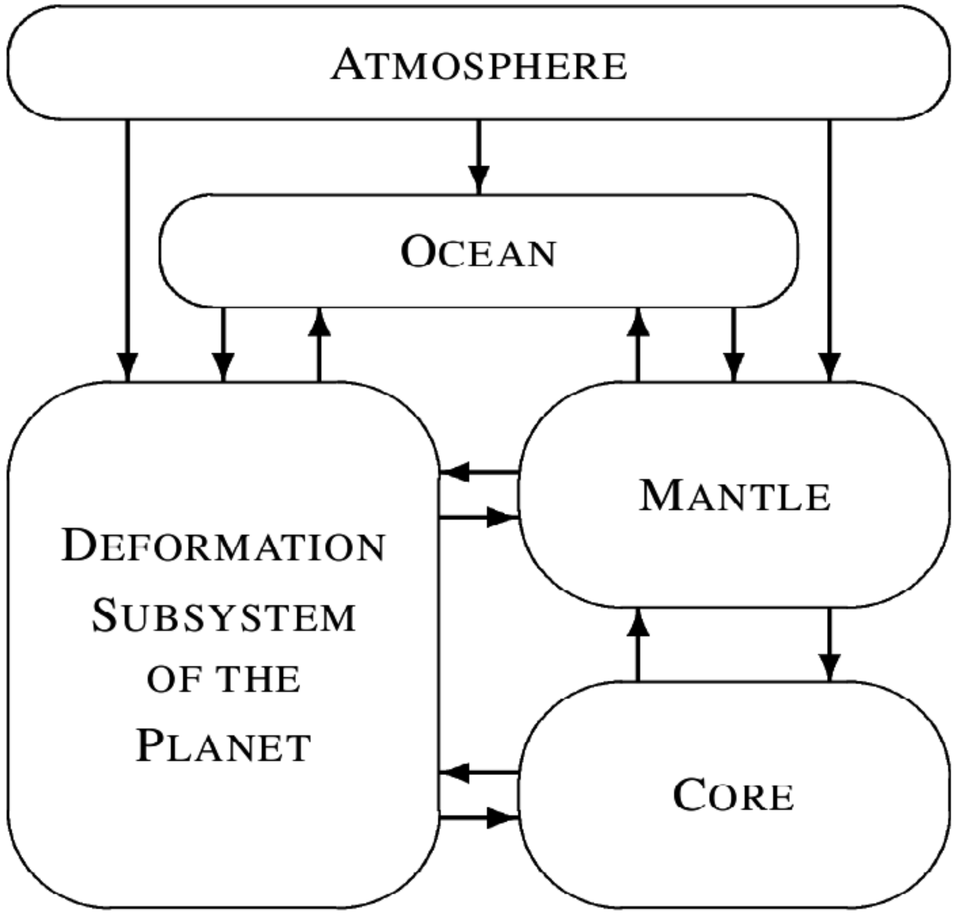
Ocean & mantle: angular momonetum exchange due to variations in the moment of inertia of the ocean (torque)

Core & Mantle: pressure and friction coupling (consistent with Poincaré flow) with torque depending on the flattening of the CMB and the viscosity of the fluid core.

Theory

A simple, modular Earth system model

Dynamical Integrate Modular Earth Rotation System (DIMERS)



System properties as function of parameters:

Model Parameters:

- Q core (Q_c)
- Q mantle (Q_m)
- core flattening; equal to CMB flattening (f_c)
- coupling parameter accounting for effect of flow pattern on core-mantle torque (x)

System properties:

- Period T and Q of CW
- Period T and Q of the NDFW/FCN

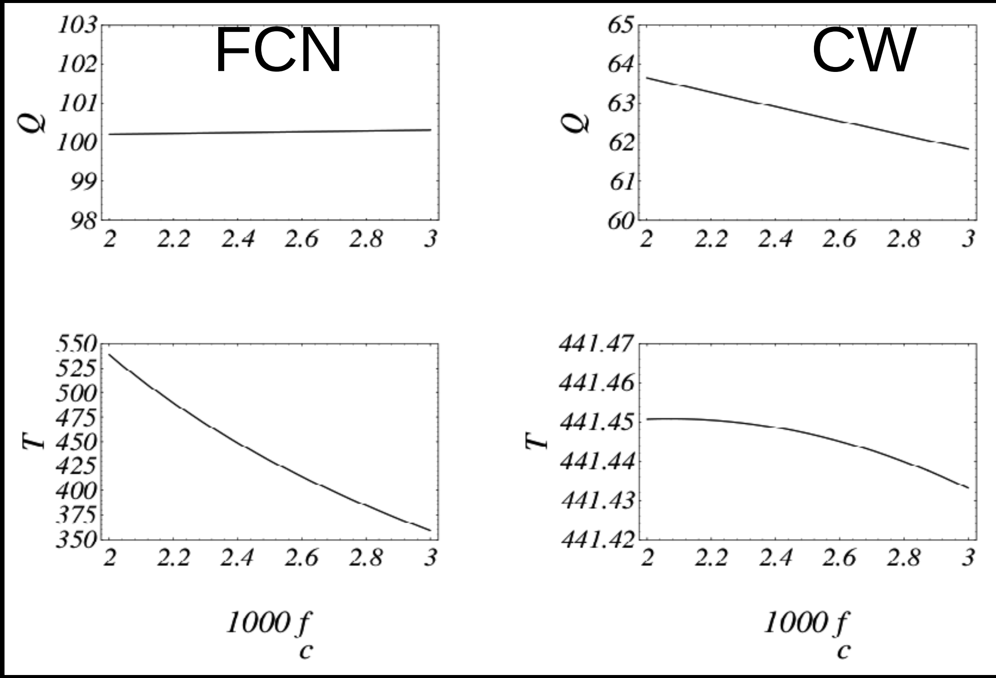
CW and FCN periods and Qs are "emerging" properties

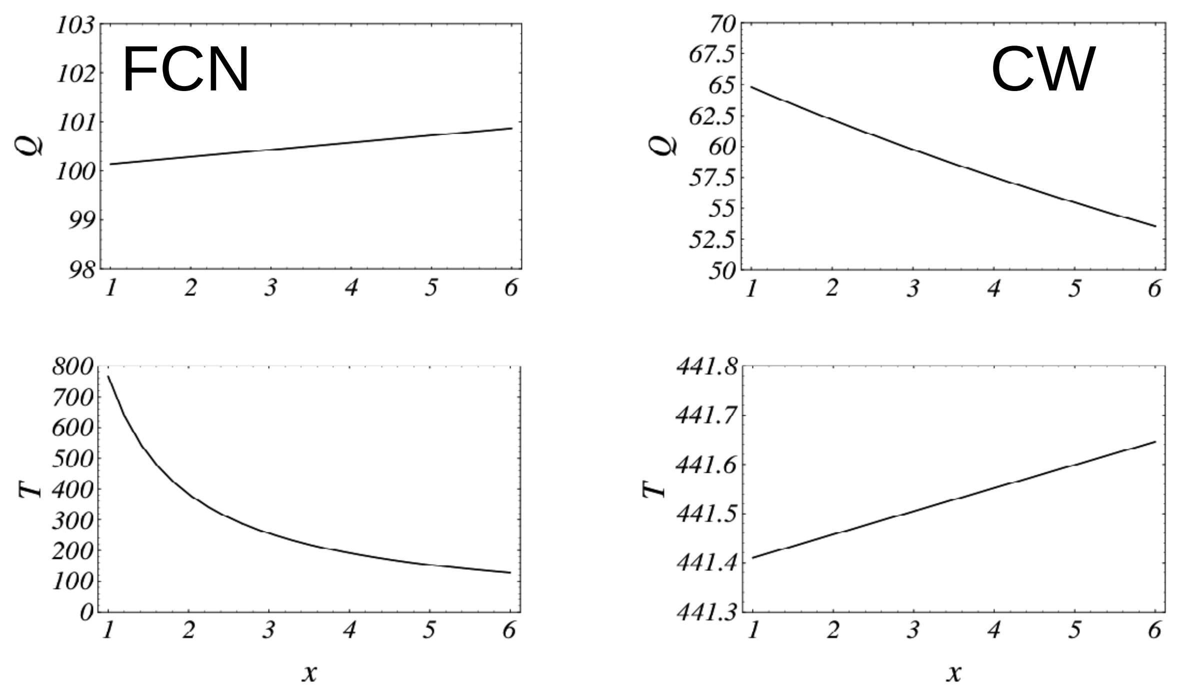
Sensitivity: FCN and CW period and Q as function of model parameters

Core flattening

FCN and CW period (sidereal days) and Q as functions of core flattening f_c

x=1.78 and Q_m=Q_c=100.





Coupling parameter x

FCN and CW periods (sidereal days) and Q as functions of the coupling parameter x

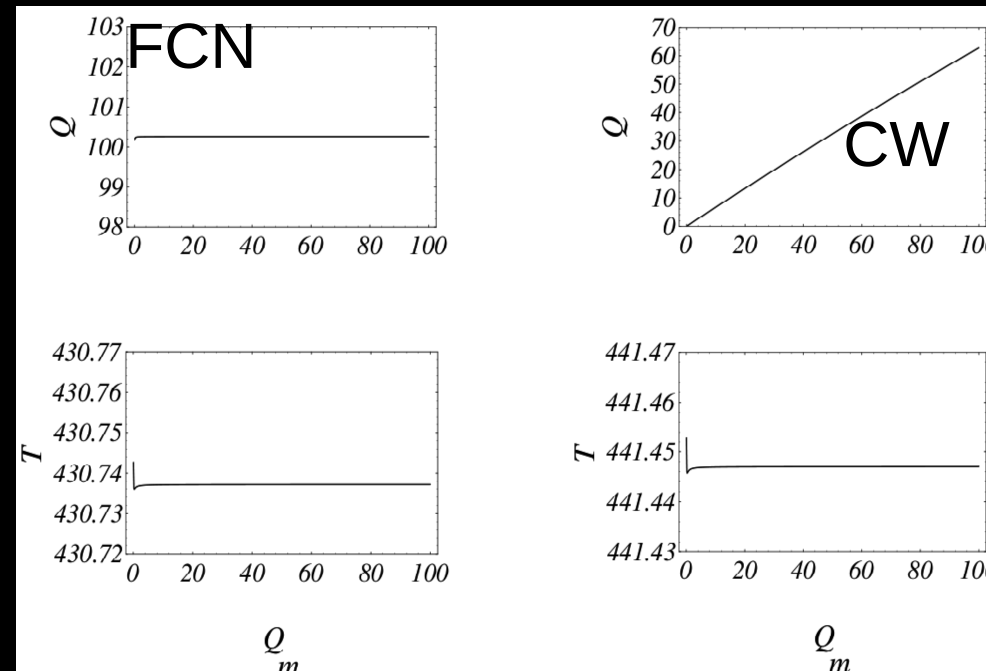
f_c=0.0025, Q_m=Q_c=100.

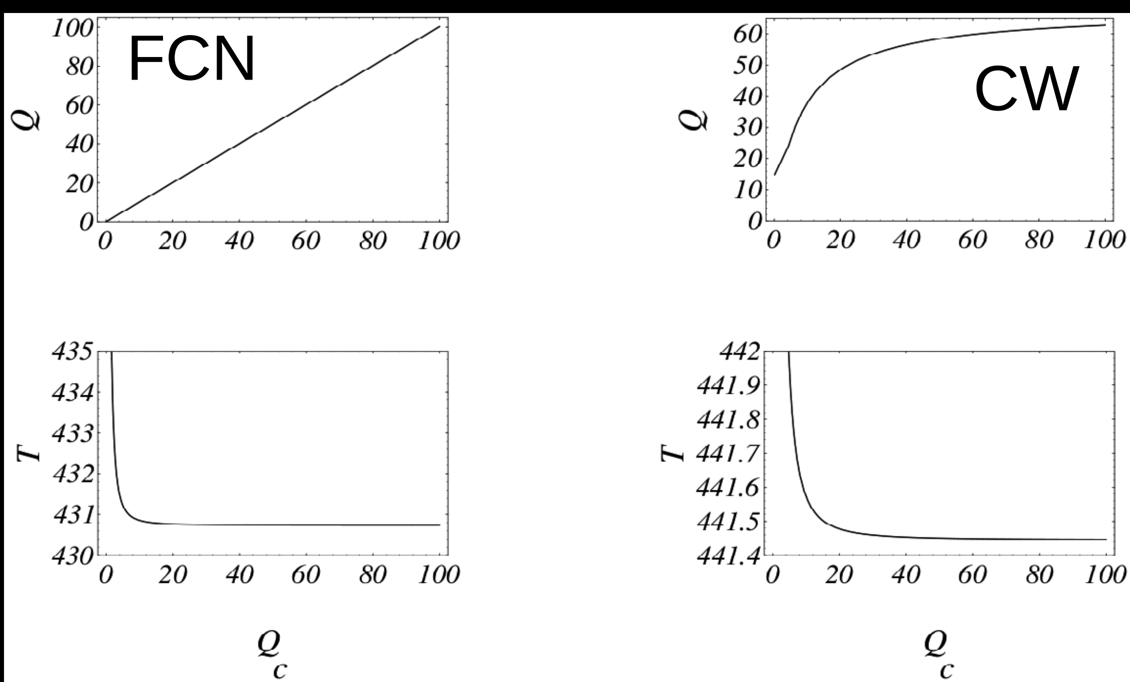
Sensitivity: FCN and CW period and Q as function of model parameters

Q mantle

FCN and CW period (sidereal days) and Q as functions of the quality parameter Q_m

x=1.78, f_c=0.0025 and Q_c=100.



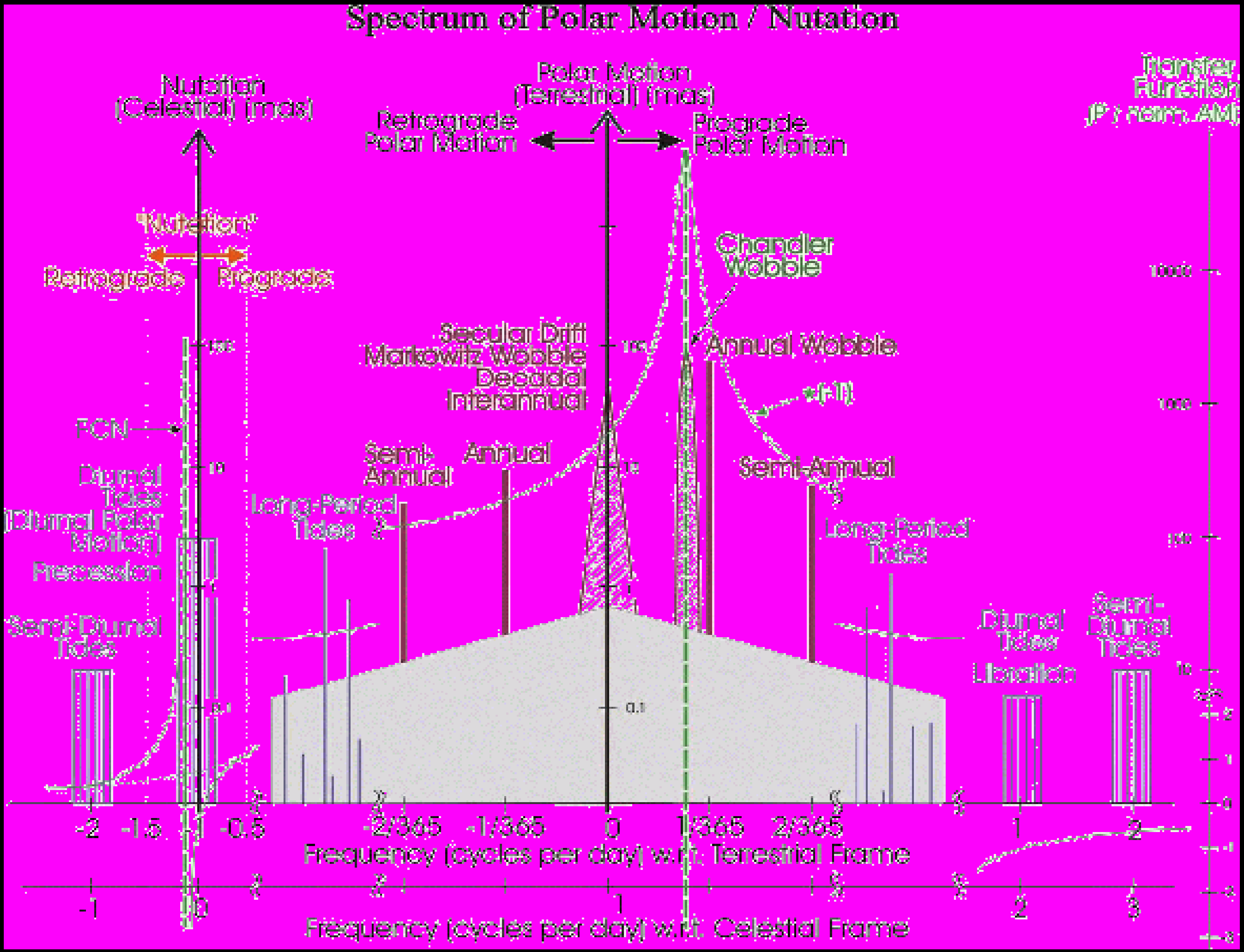


Q core

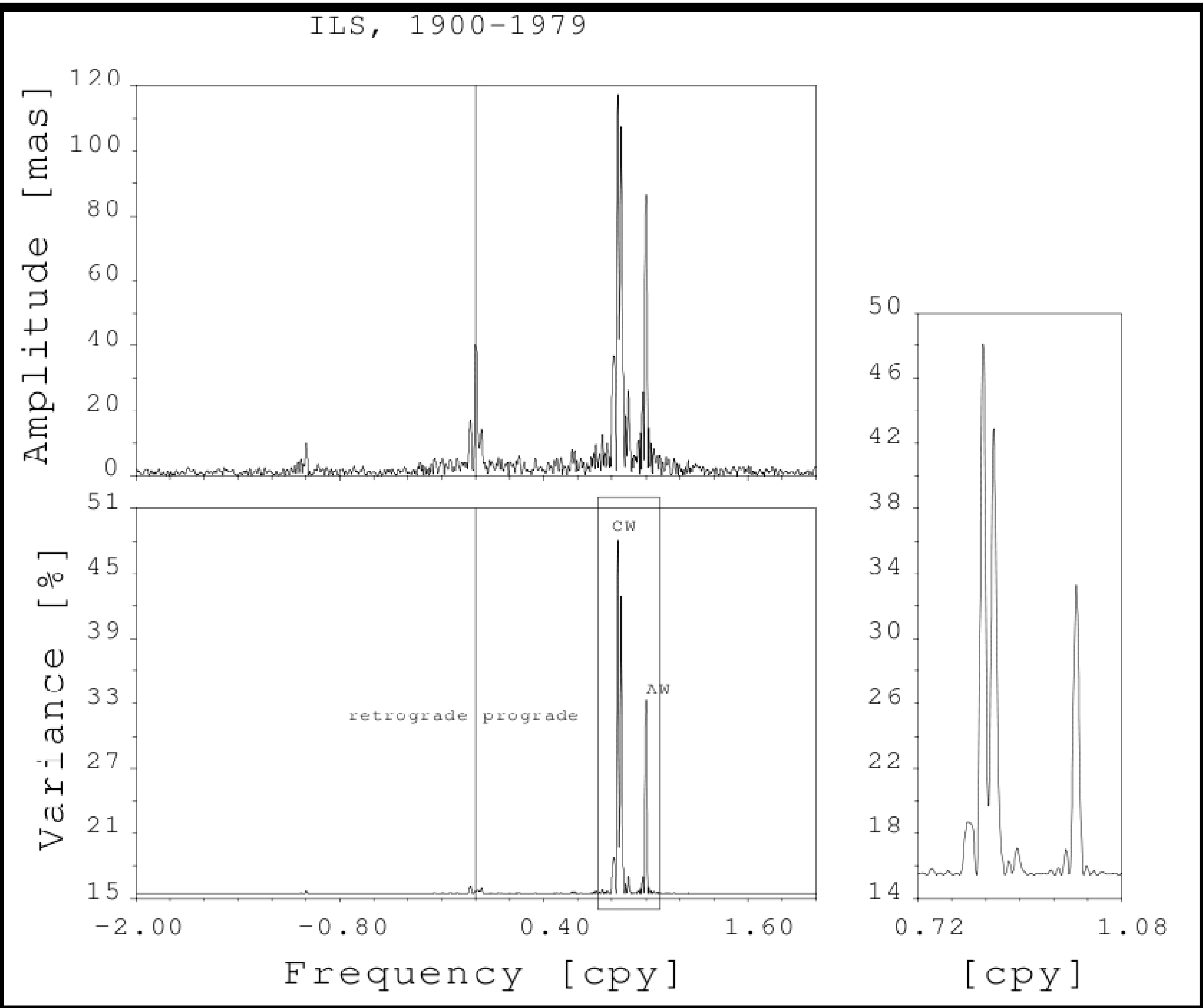
FCN and CW period (sidereal days) and Q as functions of the quality parameter Q_c

x=1.78, f_c=0.0025 and Q_m=100.

Polar Motion



Polar Motion



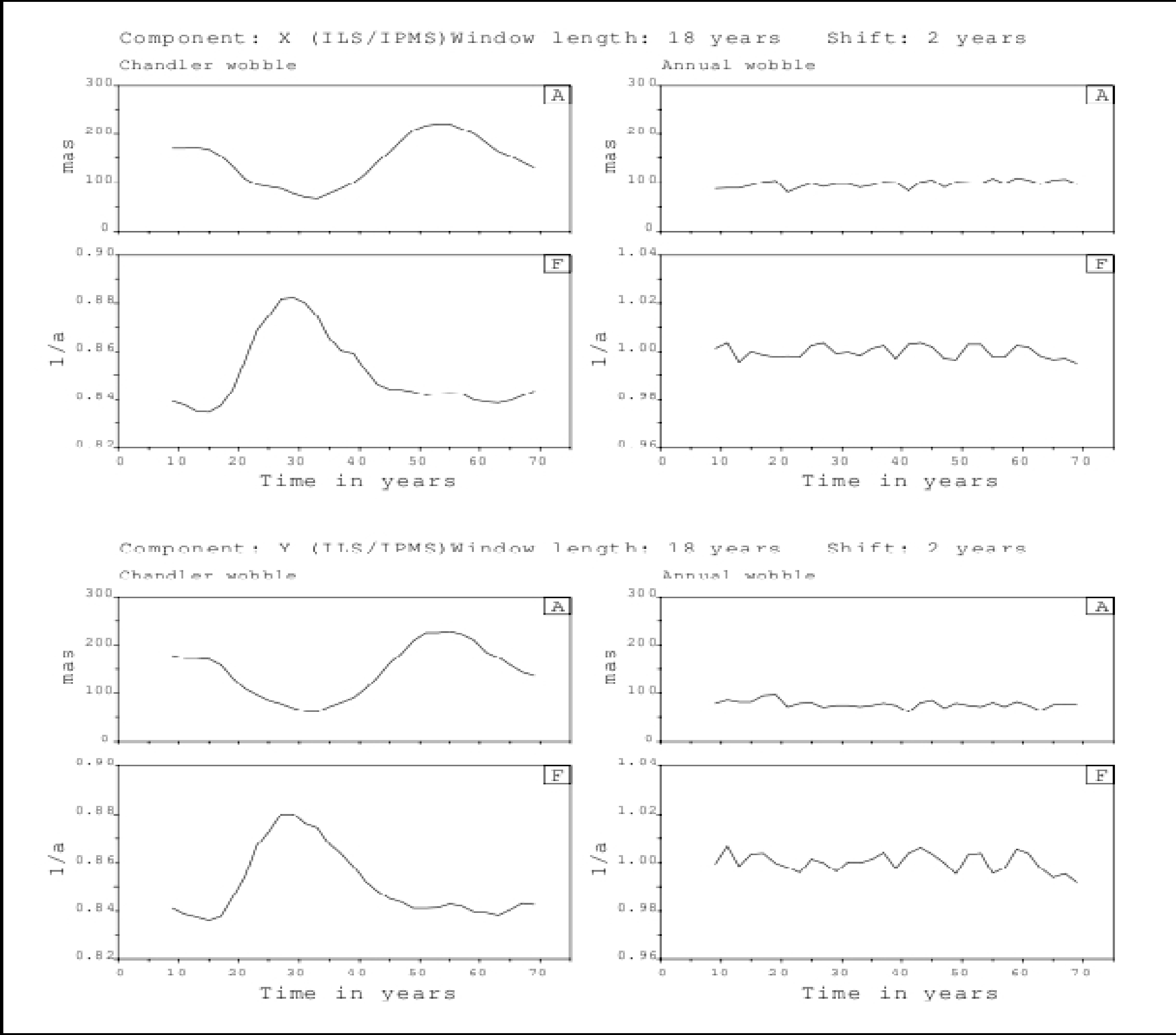
Spectrum of polar motion. Base function: circular motion with variable frequency plus linear trends in both PM components.

Polar Motion

Table 1. Some selected previous results for CW parameters.

Reference	**Main conclusions**
Colombo and Shapiro (1968)	Double peak structure from ILS data
Gaposchkin (1972)	Four peaks in the Chandler band
Guinot (1972)	Temporal variations in period, amplitude, and phase
Currie, Robert G. (1974)	One broad peak in ILS/IPMS data, $T_w = 432.95 \pm 1.02$ d, $Q_w = 36 \pm 10$
Graber (1976)	One peak with $T_w = 430.8$ d, $Q_w = 600$ from 15 yrs of IPMS data
Ooe (1978)	One peak with $T_0 = 0.8400 \pm 0.0039$ cpy and $50 < Q_w < 300$
Wilson and Vicente (1980)	Best estimates of $T_0 = 0.843$ cpy and $Q_w = 170$
Carter (1981)	Temporal variability attributed to frequency modulation due to solid Earth/ocean interaction
Okubo (1982)	Stable Chandler period, $50 < Q_w < 100$
Chao (1983)	Two major and two minor constituents in the YY series, Q ranging from -1930 to $+700$
Vondràk (1985)	Non-linear relationship between frequency and amplitude
Lenhardt and Groten (1985)	Several different models for the Chandler peak, Q is estimated to be as low as 24
Wilson and Vicente (1990)	CW period of 433.0 ± 1.1 days and $Q = 179$ with a range of 74 to 789
Kuehne *et al.* (1996)	CW frequency/period of 0.831 ± 0.004 cpy/439.5 ± 1.2 days
Furuya and Chao (1996)	CW period of 433.7 ± 1.8 days and $Q = 49$ with a range from 35 to 100

Polar Motion

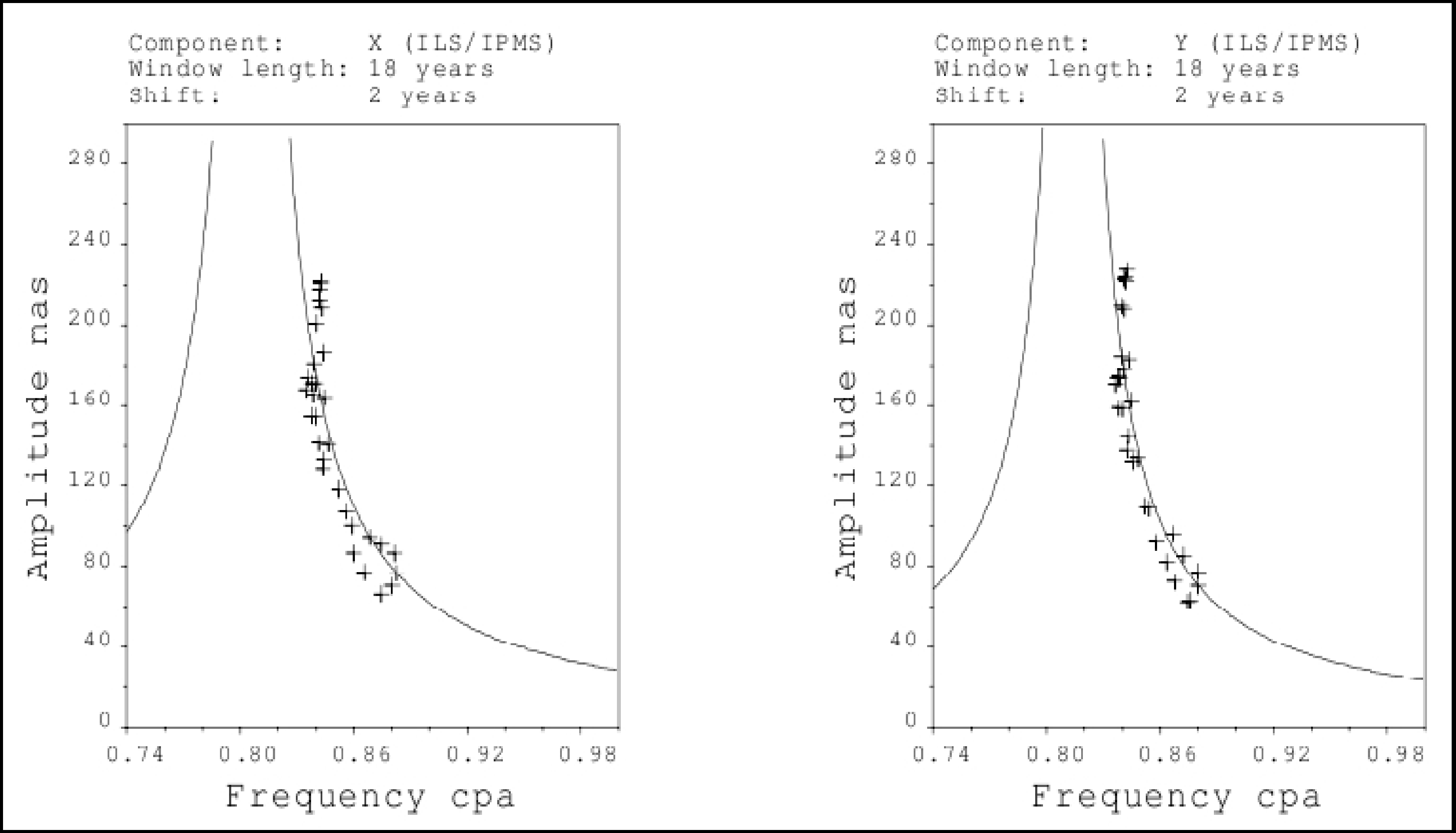


Polar Motion

Excitation of the Chandler Wobble:
- already Chandler noted that the apparent period of the wobble is variable;
- Newcomb stated that physics exclude a variable eigenperiod;
- the observed wobble was always consider to be a free, decaying wobble;
- excitations consider were earthquakes (too small), atmospheric wind, ocean bottom pressure, atmospheric pressure;
- discrepancies between excitation and observed wobble;
- much better consistency between annual excitation and annual wobble.

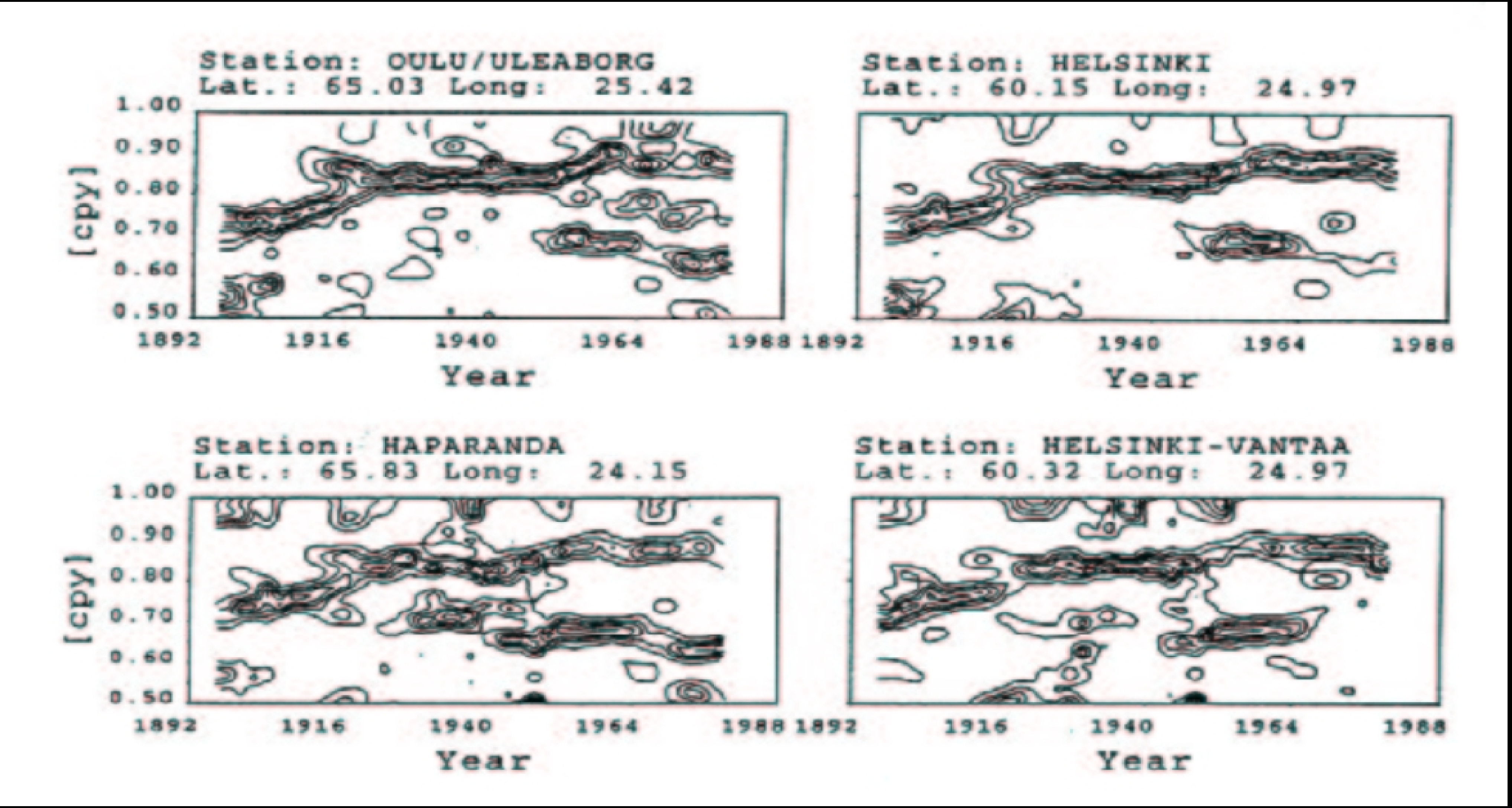
Alternative explanation (Plag, 1997): *Observed Chandler Wobble is a resonant forced oscillation (like the annual wobble) close to a (fixed) eigenperiod*.
- apparent (dominant) period of the forced wobble depends on the dominant period in the forcing;
- if the forcing period moves closer to the eigenperiod, the wobble amplitude increases.

Polar Motion



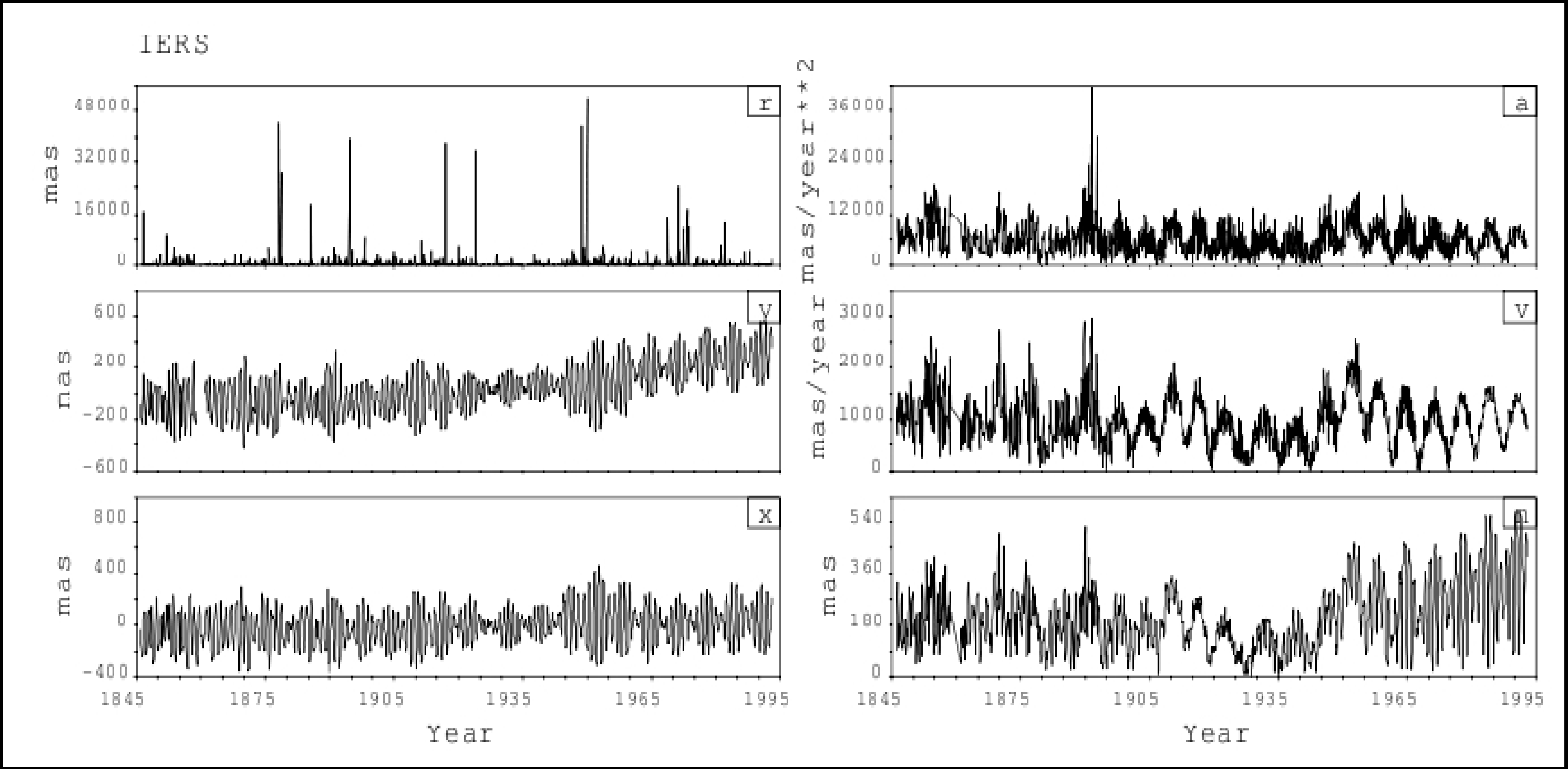
Resonance curve for polar motion (Plag, 1997). Assumption: forcing amplitude is constant, forcing period variable.

Polar Motion

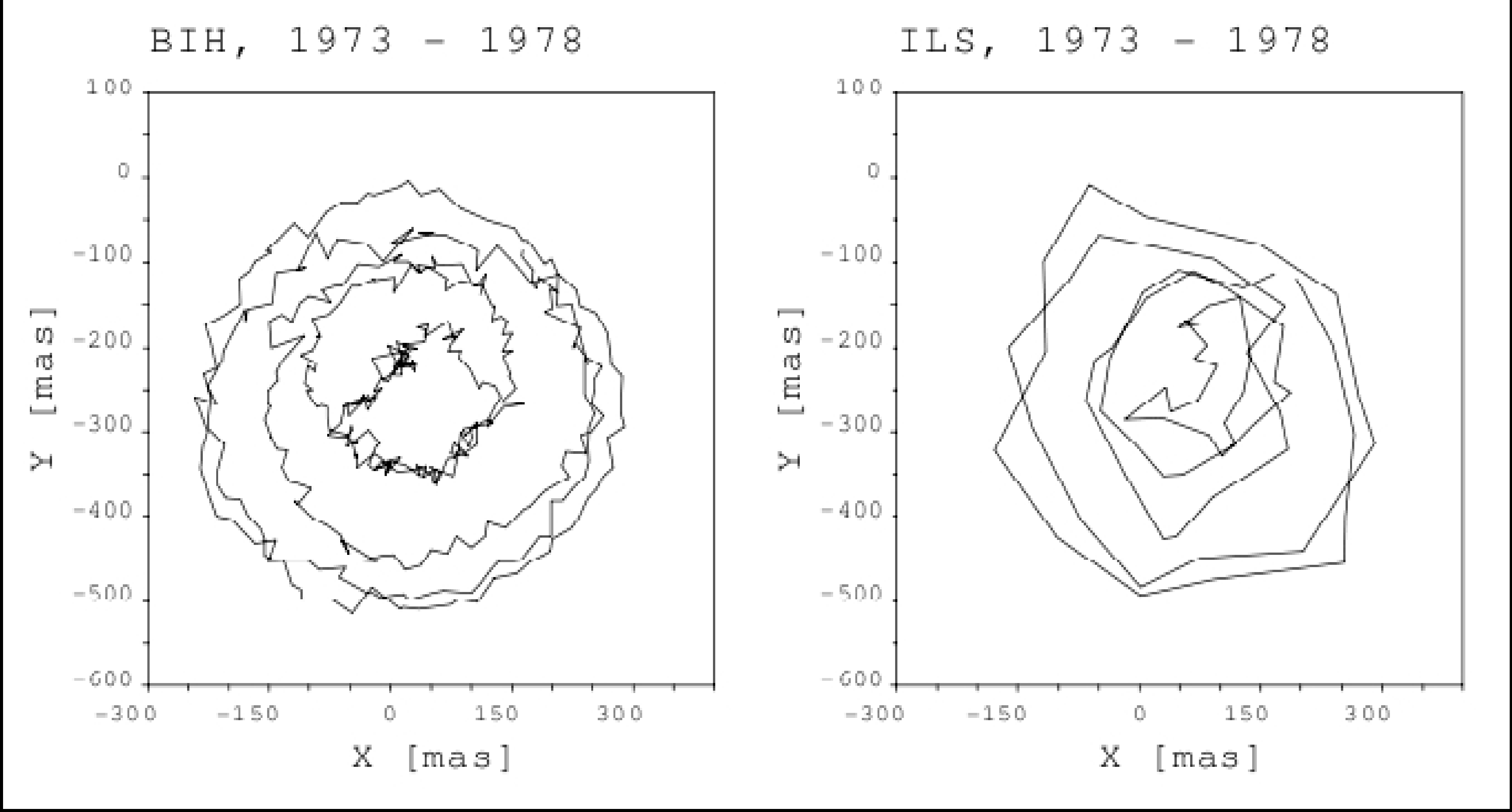


Dynamic spectrum of tide gauge records in the CW band. The apparent pole tide shows a large variability in frequency.

Polar Motion



Polar Motion



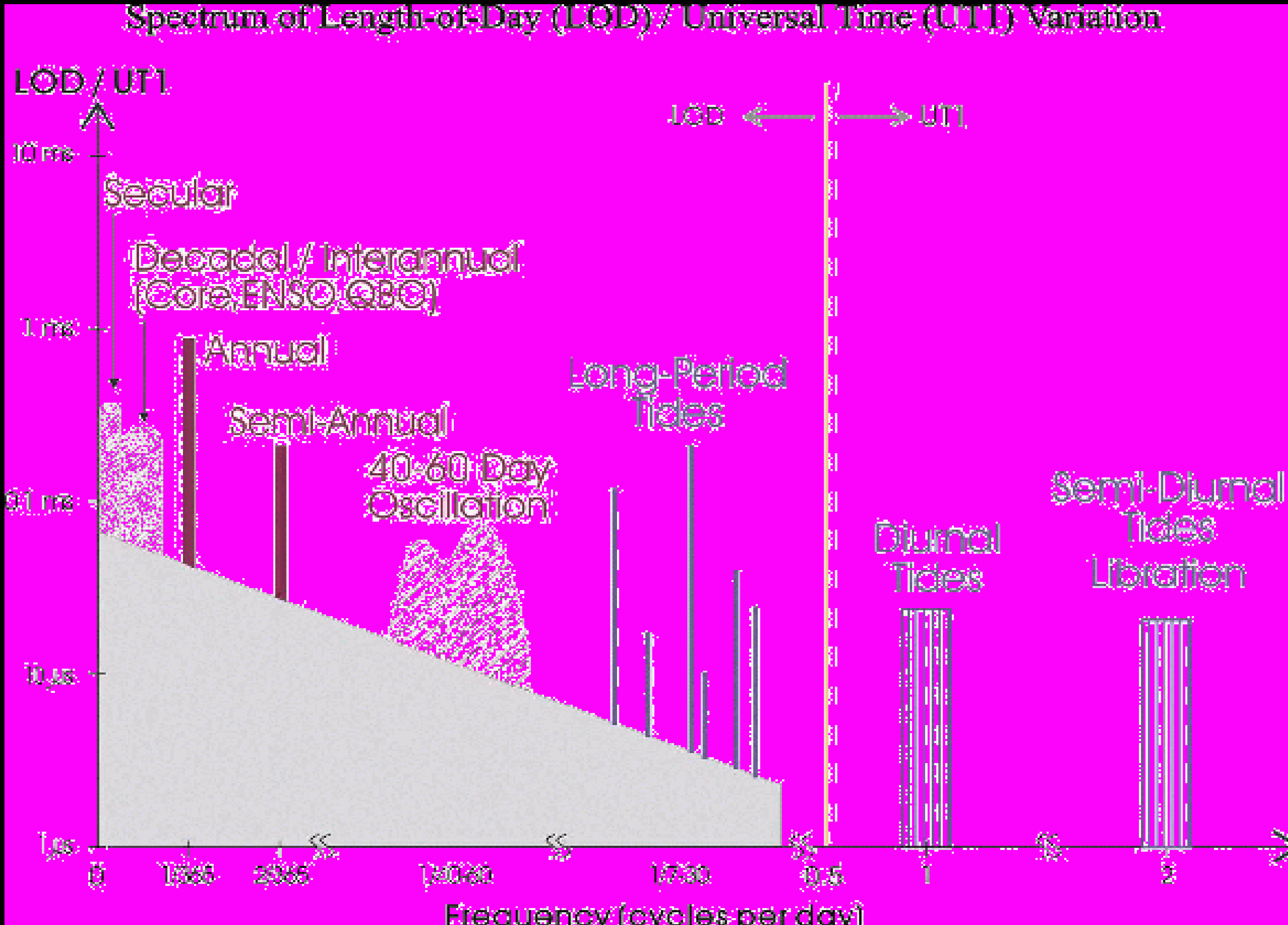
Polar Motion

Chandler Wobble is an interesting example of research trajectories:
- Euler's prediction mislead many data analyses;
- Concept of observed wobble being a 'free mode' misled search for excitation;
- data inhomogeneities raised doubts about characteristics;
- simplifications in modeling led to slow progress in understanding of the nature of the wobble;
- integration of solid Earth into a Earth system model could provide final answers.

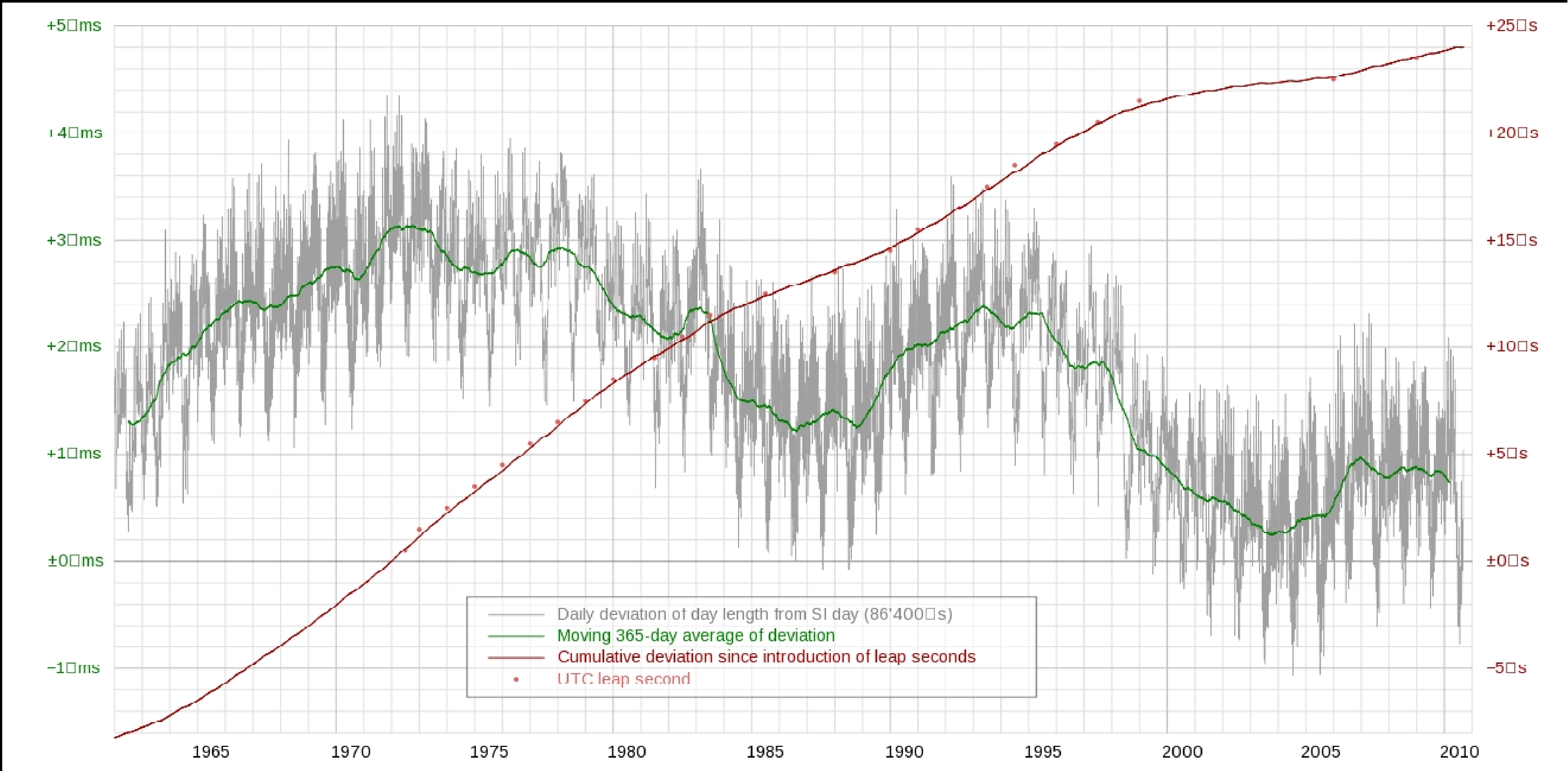
Polar Motion

Other questions related to rotation:
- **pole tide** (due to changes in centrifugal forces): equilibrium tide or dynamic? Equilibrium amplitude ~6 mm; in tide gauges up to 40 mm (e.g. in the North Sea and Baltic Sea), but temporal variation not related to CW temporal variations; Conclusion: apparent pole tide enhancement is caused by atmospheric forcing, not CW.
- **polar drift:** theoretical treatment of different groups not in agreement; drift in observations may be biased by data inhomogeneities; should be due to postglacial rebound and present-day ice melt.
- **Free core nutation**: detected in response of solid Earth to tidal forcing as an increase in transfer (admittance) function; several claims for detection in other observations (e.g. VLBI); however, theoretical studies indicate that amplitude would be far below detection levels.

Length of Day (LOD)



Length of Day (LOD)



Final Remarks