Modeling of Nutation-Precession: Very long baseline interferometry results

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Short title: Modeling of Nutation-Precession: VLBI Results

Abstract

Analysis of over 20 years of very long baseline interferometry data (VLBI) yields estimates of the coefficients of the nutation series with standard deviations ranging from 5 micro-arc-seconds (μas) for the terms with periods less than 400-days to 38 μas for the longest period terms. The largest deviations between the VLBI estimates of the amplitudes of terms in the nutation series and the theoretical values from the Mathews-Herring-Buffett (MHB2000) nutation series are 56±38 μas (associated with two of the 18.6-year nutations). The amplitudes of nutational terms with periods less than 400-days deviate from the MHB2000 nutation series values at the level standard deviation. The estimated correction to the IAU-1976 precession constant is -2.997±0.008 mas/yr when the coefficients of the MHB2000 nutation series are held fixed and is consistent with that inferred from the MHB2000 nutation theory. The secular change in the obliquity of the ecliptic is estimated to be -0.252±0.003 mas/yr. When the coefficients of the largest amplitude terms in the nutation series are estimated, the precession constant correction and obliquity rate are estimated to be -2.960±0.030 mas/yr and -0.237±0.012 mas/yr. Significant variations in the freely excited Retrograde Free Core Nutation (RFCN) mode are observed over the twenty years. During this time the amplitude has decreased from about 300±50 μas in the mid-1980s to nearly zero by the year 2000. There is evidence that the amplitude of the mode in now increasing again.

1. Introduction

Very long baseline interferometry (VLBI) measures the differential arrival times of radio signals from extragalactic radio sources. These radio sources provide the most stable definition of inertial space currently available. In a typical VLBI observing scenario, between four to eight radio telescopes, with separations of several thousand kilometers, make measurements of the differential arrival times of signals from usually 20-40 extragalactic radio sources. The radio signals from each source are recorded on magnetic tape for 1-3 minutes and later cross-correlated to determine the differential delays. During a 24-hour session, measurements in many directions in the sky are made. Each of the delay measurements has an accuracy of ~1 cm when the effects of ionospheric refraction are removed using a dual-frequency correction. Delays are measured in two different frequency bands (X-band ~8 GHz, and S-band ~2 GHz) (see, for example, *Rogers et al*. [1983]; *Clark et al*. [1985]).

The geodetic analysis of the several thousand delay measurements collected in a 24-hour period parameterizes a model for the measurements as functions of the positions of the radio telescopes, the differences in the hydrogen maser clocks used at the telescopes, atmospheric propagation delays, and the positions of the radio sources. Either least squares or Kalman filters [*Herring et al.*, 1990] are used to invert the measurements for estimates of the parameters of the model for the day of data.

One important class of the parameters determined from the analysis of the VLBI data is the Earth orientation parameters (EOP). These parameters are related to the changes in the position of the Earth's rotation axis with respect to its crust, so called polar motion, and with respect to inertial space, so called nutations [*Herring et al*., 1991]. One parameter is the related to changes in the rotation rate of the Earth and is usually expressed as the difference between Universal Time 1 (UT1) and the atomic clock time standard, Universal Time Coordinated (UTC). In this paper, we concentrate on the nutation parameters determined from VLBI measurements.

The nutations are of geophysical interest because they provide a means of studying the rotational response of the Earth to a set of periodic torques applied by the Sun, Moon, and planets. These torques are known very accurately, and the response of the Earth system to these torques is measured with high precision. The main properties of the Earth that affect the response are the presence of the fluid-outer and solid-inner cores, deformability properties of the mantle and core regions (including mantle anelasticity), and the presence of the oceans. Detailed analysis of the response allows the determination of some of the properties of these regions of the Earth [*Mathews et al*., 2001]. In particular, the nutations allow study of the properties of the inner fluids of Earth that are difficult to study be other means. The largest effect of the non-rigidity of the Earth arises from the presence of the fluid core and its interaction with the mantle. The differential rotation between these two regions of the Earth results in a resonance in the Earth's rotation with a nearly diurnal period. We refer to this resonance as the retrograde free core nutation (RFCN). When viewed from inertial space, this resonance has a retrograde period of about 430 days. In addition, the differential rotation of the solid-inner core introduces another resonance with a prograde period, in inertial space, of about 946 days referred to as the prograde free core nutation (PFCN) [*Mathews et al.,* 1991; *Mathews et al.,* 2001].

The normal method for determining the nutations of the Earth is to first compute the nutations of a rigid body with the same dynamical ellipticity as the Earth. These rigid-Earth nutations are then convolved with a frequency-dependent transfer function that gives the response of a more realistic representation of the Earth. Due to the periodic nature of the orbits and the perturbations of the orbits of the Earth, moon, and planets, the nutations of the rigid Earth are expressed as the sum of a series of periodic terms whose arguments are based on angles that represent the positions of the celestial bodies. This type of expansion can then be easily convolved with the frequency dependent response function.

The currently adopted standard theory for the nutations of the Earth is the IAU-1980 nutation series [*Seidelmann*, 1982]. The rigid Earth series used contained 106 terms with terms greater than 0.1 milli-arc-seconds (mas) and the coefficients were truncated at 0.1 mas [*Kinoshita*, 1977]. This series was convolved with a transfer function for an ellipsoidal, elastic Earth with fluid-outer and solid inner core [*Wahr*, 1981]. Although the solid-inner core was included in the calculations for this transfer function, it had no direct effect on the transfer function. Analysis of VLBI data in the mid-1980s quickly revealed deficiencies in this theory arising mostly from the non-hydrostatic shape of the core-mantle boundary [*Herring et al.,* 1986; *Gwinn et al.,* 1986]. Developments over the next decade increased the accuracy of the rigid-Earth nutation theory [*Bretagnon et al*., 1997; *Bretagnon et al*., 1998; Souchay *and Kinoshita*, 1996, *Souchay and Kinoshita*, 1997; *Souchay et al*., 1999; *Roosbeek and Dehant*, 1998] and the completeness of the transfer function [*de Vries and Wahr*, 1991; *Mathews et al*., 1991*a*, *Mathews et al*., 1991*b*].

In this paper we use the Mathews-Herring-Buffett (MHB2000) nutation series [*Mathews et al*., 2001] as the series to which the VLBI results are compared. This series is generated by the convolution of the transfer function from Mathews et al. [2001] with the rigid-Earth nutation series REN-2000 [*Souchay and Kinoshita*, 1996; *Souchay and Kinoshita,* 1997; *Souchay et al*., 1999]. We used the expressions for the arguments of the Sun, Moon, and planets from Simon et al. [1994]. After merging terms with identical arguments, the Luni-solar part of the nutation series contains 678 terms with amplitudes larger than 0.1 micro-arc-seconds (μas), and the planetary part contains 687 terms.

In addition to the forced nutations represented by MHB2000, there can also exist nutational type motions from the free excitations of the RFCN and PFCN modes. These motions are analogous to the Chandler Wobble in polar motion [*Gross and Vondrak*, 1999]. Excitation of a normal mode will generate a free nutation with a period equal to the eigenperiod of the mode and with a complex amplitude which can change with time depending on the variations in the excitation. For the RFCN mode, atmospheric pressure variations seem to be the most likely source of excitation [*Sasao and Wahr*, 1981]. Previous analyses of VLBI data have detected the freely excited RFCN with amplitudes between 100 and 200 μas [*Herring et al*., 1991; *Herring and Dong*, 1994]. In this paper, we consider this problem further by examining the temporal variations in this freely excited mode. The PFCN resonance is so small relative to the RFCN resonance that any free excitation of this mode is not likely to be detected with current measurement accuracy.

In this paper, we first discuss the analysis of the nutation angle data sets obtained from VLBI measurements and we present the results from the complete analysis. We then discuss the details of error analyses performed on these results and the temporal evolution of the freely excited RFCN.

2. Data Analysis and Results

The analyses in this paper use two sets of measurements of the nutations of the Earth both obtained from similar sets of VLBI experiments. One analysis by the Goddard Space Flight Center, obtained from the International VLBI Service (IVS) products area (ftp://cddisa.gsfc.nasa.gov/vlbi/ivsproducts/eops), included results from 2974 sessions of data, each normally of 1 day duration, collected between August 1979 and November 1999. This series is referred to as GSF1122. The other analysis, obtained from the U.S. Naval Observatory, referred to as usn9901, covered a similar interval of time and included 2713 sessions of data. The GSFC and USNO analyses differed in the models used for diurnal and semidiurnal Earth rotation variations; and the spans of time used to estimate atmospheric delay parameters. In addition to these results, we also used an analysis by the Institute of Applied Astronomy (IAA). The IAA analysis used data collected between January 1984 and July 2000, and includes only the 981 VLBI measurements made for routine EOP determination. For additional checks on the results, we also used data from January to July 2000 from the GSFC and USNO analyses. These data from 2000 were not used in the standard analysis but rather were used as an independent data set to evaluate the results from the pre-2000 data. All of these data sets are available from the IVS.

The data sets consist of a time (at the center of the VLBI session) and differences between the measured nutations in longitude, $\Delta\psi$, and obliquity, $\Delta\varepsilon$, and the IAU-1980 theoretical values for these nutation angles. Each pair of nutation angles has standard deviations derived from the geodetic analysis of each session. These standard deviations are consistent with the χ^2-per-degree of freedom (χ^2/f) of the VLBI delay measurements being unity for the session being analyzed. The data sets are shown in Figure 1. Although the full correlation matrix is available for the GSFC analysis, we did not use these correlations because, as shown in Herring et al. [1991], they make little difference to the type of analysis performed here. In our analysis of these data we seek to obtain estimates of the differences between these nutation angles and those inferred from a modern nutation series, and to estimate corrections to the largest terms in the nutation series. For this estimation we need appropriate standard deviations for the nutation angles estimates.

Although the standard deviations of the nutation angle estimates for a day are in accord with the scatter of the VLBI delay residuals on that day, analysis of the angle residuals shows that these standard deviations are most likely too small. We use here the procedure adopted by Herring et al. [1991] to determine more realistic standard deviations. We binned the nutation angle residuals by the size of the standard deviation and then computed the weighted-root-mean-square (WRMS) scatter of the nutation angle residuals in each bin. To these binned values, we fit a model of the form

$$\sigma_i^2 = \sigma_o^2 + \beta\sigma_i^2 \qquad (1)$$

where $\square_i$ is the WRMS scatter in the *i*th bin, $\square_o^2$ is a constant additive variance, and $\square$ is a scaling of the expected scatter of the residuals in the bin $\square_i^2$. The nutation angle residuals used in this process are obtained from fitting nutation series parameters to the data and therefore depend on the standards deviations assigned to the angle data. The fitting of the parameters is iterated and new residuals computed after initial estimates of the parameters of Equation (1) are determined. The iteration converges after the second iteration. Since parameters are estimated, the residuals are almost independent of the apriori nutation model used in this analysis. The final fit to this model yields for both $\square\square$ sin $\square_o$, where $\square_o$ is the mean obliquity of the ecliptic, and $\square\square$ values of approximately $\square_o^2$=(0.08 mas)^2 and $\square$=1.6. The binned WRMS scatter results and the model fit are shown in Figure 2. These values are considerably less than those reported in Herring et al. [1991] ($\square_o^2$=(0.34 mas)^2 and $\square$=2.1) which is probably due to improved modeling of the VLBI delay data themselves, improved error models in the VLBI analysis, and the completeness of the apriori nutation series used here. Analysis of the nutation angle residuals from the time interval used in Herring et al. [1991] (July 1980-February 1989) yields error-model parameters of $\square_o^2$=(0.16 mas)^2 and $\square$=1.3, showing that the standard deviations are now more in accord with the WRMS scatter than at the time that Herring et al. [1991] was published. In the error analysis section we discuss additional modifications needed to generate realistic uncertainties for the coefficients of the nutation series.

The IAU-1980 series in not adequate for determining corrections to the nutation series due to the truncation level and the large number of missing terms. For the analysis here we have adopted the MHB2000 nutation series [*Mathews et al*., 2001] as the apriori theory to which corrections are estimated. In addition to the 678 frequencies included in the luni-solar terms in this theory, we also include 687 planetary nutation frequencies from REN-2000 [*Souchay et al*., 1999]. The planetary contributions are shown in Figure 3.

The MHB2000 nutation series is not independent of the GSFC and USNO data sets discussed here. The corrections to terms in a given nutation series are used to determine the "best-fitting Earth parameters" (BEP), such as the dynamic flattening of the fluid core and core-mantle coupling constants, as discussed in Mathews et al., [2001]. A new nutation series is computed based on the estimated BEPs and new corrections are computed which are used to further refine the estimates of the BEPs. This scheme converges in just two iterations when the theoretical basis of the nutation series is not changed..

We perform two classes on analyses on the data sets discussed above. In the first class, which we refer to as the "amplitude" analysis, we estimate the corrections to complex amplitudes of the 21-frequencies in the nutation series. The specific terms chosen are those that could be reliably estimated, i.e., some of the nutation frequencies are so close that separate estimates could not be reliably obtained. For each frequency, four coefficients are estimated representing the prograde and retrograde frequencies, and the in- and out-of-phase components. In addition to these 84 components, we also estimated secular trends in the nutation angles (corresponding to a change in the precession constant and the mean rate of change of the obliquity of the ecliptic), and time-dependent freely excited RFCN amplitudes. In the other class of analysis, referred to as the "series" analysis, we estimate only the time varying RFCN terms and the secular terms. We adopt as known all of the forced terms in MHB2000 nutation series. In this latter class of analysis, we are most interested in the temporal changes of the RFCN mode. In all analyses, constant offsets are estimated for $\Delta\psi \sin \epsilon_0$ and $\Delta\epsilon$ to allow for re-orientation of the celestial reference frame to the J2000 frame realized by the nutation series.

The temporal resolution of the estimates of the RFCN free mode is different for the amplitude and the series analyses. In the amplitude analysis, near unity correlations occur if the time resolution of the RFCN amplitudes is too short. Essentially, the time variable RFCN amplitudes can mimic the behavior of other periodic terms. In the amplitude analysis we estimate the RFCN in six time intervals whereas in the series analysis ten intervals are used. In some trial analyses discussed below we use even more intervals.

The series analysis using the combined GSFC and USNO data set is referred to as MHB2000 and uses ten intervals for the RFCN free mode. The data sets were combined by concatenating the two data files. Analysis of the differences between the two data sets showed that the mean differences were small (20 μas and 30 μas for $\Delta\psi \sin \epsilon_0$ and $\Delta\epsilon$, respectively) as should be expected given the common celestial reference definition used in the two analyses. The GSFC analysis contained 317 estimates not in the USNO analysis and the USNO analysis contained 61 estimates not in the GSFC analysis. The WRMS differences between the data sets were 157 and 169 μas for $\Delta\psi \sin \epsilon_0$ and $\Delta\epsilon$, respectively, with corresponding χ^2/f of 0.54 and 0.63.with standard deviations of each day assigned according to Equation 1. The results from this analysis are available electronically as the MHB2000 nutation series. The full nutation series and the time dependent RFCN amplitudes have been coded as a series of Fortran77 subroutines that are also available electronically at http://www-gpsg.mit.edu/~tah/mhb2000. From this analysis the estimates of $\Delta\psi/dt$ and $\Delta\epsilon/dt$ are –2.997±0.007 mas/yr and –0.252±0.003 mas/yr. The estimate of $\Delta\psi/dt$ matches the expected precession constant change computed from the dynamic ellipticity in Mathews *et al.*, [2000] to within 0.001 mas/yr.

The estimates of the luni-solar nutation amplitudes and the residuals relative to MHB2000 from the amplitude analysis are shown in Table 1. The uncertainties are twice the formal estimates computed using the error model discussed above for terms with periods less than 400-days and four times the formal estimates for the longer period terms. These scaling factors are determined empirically in the error analysis section and are consistent with a reddened error spectrum. For the shorter period terms, half of factor-of-two multiplier is to account for the double use of the VLBI nutation angle estimates, i.e., both the GSFC and USNO data analyses use very similar data sets. The remaining factor is to account for temporal correlations between the VLBI nutation angle estimates as discussed in the next section. The error analysis section also shows that longer period terms have even larger errors and so we have increased the scaling factor by another factor of two for these terms. The WRMS scatters of the nutation angle residuals from this analysis are 183 and 189 μas for $\Delta\psi \sin \epsilon_0$ and $\Delta\epsilon$, respectively. As expected based on the error model used χ^2/f of the residuals is close to unity (0.917 and 0.943 for the two components). From the series analysis, in which the coefficients of the series are fixed, the WRMS scatters of the nutational angle residuals for $\Delta\psi \sin \epsilon_0$ and $\Delta\epsilon$ are 188 and 194 μas with χ^2/f of 0.975 and 0.994, respectively.

Large differences between the MHB2000 nutation series and the VLBI results shown in Table 1 appear to occur only for terms with periods greater than 400 days. The χ^2/f for the 64 estimates with periods less than 400 days is 1.08 (after scaling by the factor of two) while for all 84 estimates the χ^2/f is 2.30 (when a frequency independent scaling factor is used). Figure 4 shows the χ^2/f for the amplitude estimates as a function of the longest period term included in the χ^2 calculation. The increase in χ^2/f when the longer period terms are included can be seen. The χ^2/f for the five longest period terms is 4.07. This behavior motivates our further re-scaling of these standard deviations. There is also an indication that the shortest period terms may fit too well although the number of degrees of freedom is small for these terms (only 16-estimates are used in the statistical calculation). It is likely that correlations between the nutation angles estimates will have a larger effect on the longer period terms. We explore this possibility in the error analysis section below.

The estimates of the freely excited RFCN resonance and its temporal variations are clearly significant in this analysis. Herring et al. [1991] tentatively concluded that the free mode had been detected and estimated the complex amplitude to be (160-210*i*)± 40 μas during the 1979-1989 interval. (We represent the cosine and sine terms as a complex number.) The phase of the term was set with zero phase on Jan 1.5, 2000 which is the same convention used in this paper. *Herring and Dong* [1994] estimated the components of the free mode to be (72-158*i*)± 15 μas for data collected between 1984 and 1992.5. Our time dependent estimates for both the freely excited RFCN mode and the prograde annual nutation are shown in Figure 5. The latter is included for comparison purposes and because this term could be driven by the S1 atmospheric thermal tide whose phase and amplitude may change with time.

We modeled the variations in the RFCN mode with a piecewise linear function, which is defined to have linear variations between "nodes" at selected times. The estimated parameters are the complex amplitudes of the RFCN mode at the times of the nodes. The variation in the prograde annual term was, on the other hand, modeled as a piecewise constant function where the average value over selected intervals of time are estimated. The reason we treat these two processes differently is that the variations in the RFCN appear significant, whereas for the prograde annual term the complex components are relatively constant. In trial analyses, we treated both as piecewise constant functions and the results are consistent with Figure 5 in that the average values of the RFCN lie on the interpolation between the node values.

Using the piecewise linear function for the RFCN does introduce some problems. For the amplitude analysis, shown in Table 1, where the nutation amplitudes are estimated in addition to the time variable RFCN mode, there are large correlations between the time dependent RFCN parameters and the amplitude estimates. These correlations can be reduced greatly by increasing the time between the nodes in the piecewise linear function. For this type of analysis, we estimated the amplitude at only six nodes. However, even with this small number of nodes, there is a strong correlation between the estimate of out-of-phase retrograde 386-day nutation and the time dependent terms. The largest correlation in 85% which increases the standard deviation of the retrograde 384-day period term by a factor of four over the prograde amplitude. The out-of-phase retrograde 384-day nutation has an amplitude of only 4 μas in the nutation series and so we constrained its estimate with a standard deviation of this size. The choice of times for the nodes in the piece-wise function was based on the more frequent estimates obtained from the series analysis in which the correlations are greatly reduced because the amplitudes of individual nutation terms are not estimated. There is a potential problem with this approach in that errors in the coefficients in the nutation series could alias into a time variable RFCN mode. The high correlations between the retrograde annual and retrograde 386 day nutations and the time dependent RFCN amplitudes indicates that this possible. We do not believe that this is happening to any significant degree because the adjustments to the retrograde 386 nutation are small when piecewise constant function is used in a separate analysis as the time dependent model, and this nutation is more affected by these correlations than the retrograde annual nutation. We conclude that during the last 20 years there has been a significant change in the amplitude of the RFCN free mode.

The most likely origin for the excitation of RFCN free mode is atmospheric pressure variations. As shown by Sasao and Wahr [1981], the P_{21} spherical harmonic component of atmospheric pressure changes can efficiently excite the RFCN free mode. The S1 thermally driven tide, which can be clearly seen in the atmospheric angular momentum data, seems to contribute significantly to the prograde annual nutation [*Dehant et al.*, 1996; *Gegout et al.,* 1998]. The spectral peak at the S1 tide is large and the continuum power across the diurnal band appears large enough to drive the RFCN free modes to the amplitudes observed. It is not clear whether the variations in the continuum are large enough to explain the variations seen in the free mode estimates. In principle, the currently available atmospheric angular momentum data sets with 6-hour time resolution could be used to compute the expected variations in the RFCN free mode. We are currently investigating whether the atmospheric angular momentum (AAM) determined by the world's meteorological services is sufficiently accurate to allow this calculation.

The existence of the time variable RFCN poses two problems. For very precise astrometry that requires sub-milliarcsecond knowledge of the orientation of the Earth in space, the time variable RFCN mode will need to be monitored in much the same way polar motion and UT1 are monitored. Supplying regular updates to the amplitude of the RFCN free mode should be one of the prime functions of the IVS. However, unlike polar motion and LOD which exhibit large variations over a wide spectral range, the non-predictable part of the nutations seems to be restricted to a very narrow frequency range suggesting that it can be monitored with occasional measurements. The other problem posed by the time variations in the RFCN free mode is that it is likely to limit the accuracy with which the RFCN resonance parameters can be determined. In Table 1, the standard deviation of the estimates of the adjustments to the retrograde annual nutation, the nutation most effected by the RFCN resonance, are twice the size of the other short-period nutation terms. The inflation of its standard deviation is totally dependent on the number of RFCN nodes estimated which, in turn, is dependent on how rapidly the RFCN mode can change. From Figure 5, it appears that interpolation between nodes separated by up to 5 years yields an adequate representation but there are some year-to-year variations, which might represent real variations in the amplitudes. If the AAM data sets are of sufficient accuracy and the atmosphere is the only major source of excitation, then characterizing the frequency with which new estimates of the RFCN amplitude will need to be made should be possible. If there are large variations over just a few years, then reliably estimating the RFCN resonance parameters with an accuracy much better than is available now, will be difficult unless the free mode amplitudes can be computed from the AAM data sets.

Error Analysis

We now discuss in more detail the analysis of that nature of the error sources in the results presented in this paper. For this analysis we will follow procedures similar to those in Herring et al. [1991] and Herring and Dong [1994] in that we divide the nutation angle data in different ways and develop a statistical model which is consistent with the changes in the results seen with different divisions. In this paper, we have an additional consideration in that the primary data set used is the combination of two analyses of very similar data sets. These two analyses uses the same software, CALC/SOLVE, and so the differences between the results should be due only to the subjective decisions made by analysts while processing data. We also examine another analysis of the VLBI data using a different analysis program in this case the OCCAM program [*Titov and Zarraoa*, 1997]. In this case differences in the nutation angle estimates could arise from differences in the theoretical delay and statistical models used in the software.

Firstly, we consider the differences between the three data sets we have used and how well these data sets can be fit to a model of nutation. Table 2 gives two types of statistics. One group is the statistics of nutation angle residuals from the amplitude analysis, the series analysis, and the MHB2000 nutation series. The other group is the statistics of the differences between the data sets. (This latter comparison is independent of any specific nutation theory.) The WRMS scatter of the residuals to the any of the fits to the nutation series ranges between 180 and 205 μas, and the WRMS scatter of the differences between the data sets are of the same magnitude. The difference between the USNO and IAA data sets is larger than the fit of either of them to any of the amplitude analyses. Even between the two data sets that use the same software (USNO and GSFC), the WRMS differences are only slightly smaller (157 and 172 μas) than their individual fits to the nutations series. It is for this latter reason that the standard analyses use the merged GSFC and USNO data sets. The difference between these two data sets is not small and we have no reason to believe one data analysis is superior to the other.

One method of evaluating the quality of the estimated parameters from any data set is to compare results from subsets of data. The two usual problems with this type of analysis are that (a) correlations between measurements can give deceptively good agreement between the subsets if the correlations are not accounted for, and (b) the divided data sets yield larger standard deviations and hence the accuracy of the full data set can not be assessed. We performed a number of different divisions of the data. The general character of the results can be seen in Figure 6. Here we show results from two styles of comparisons: (a) division of the data set from a single analysis center and (b) comparison between analysis centers. Similar to the results shown in Figure 4, most tests of the division of data showed agreement between the nutation amplitudes with periods less than 400 days. In Figure 6 we show two cases for the difference between the GSFC and USNO analyses, and the difference between the IAA analysis and the combined GSFC and USNO analyses. The IAA differences, which use different analysis software from the GSFC and USNO analyses, are larger than the USNO/GSFC differences, but are not that much larger, indicating the algorithms used in the theoretical models for the two programs are quite similar. For these two comparisons, the χ^2/f for the long period terms are large again showing that the longer period terms are not as well determined as the short period nutations. This same conclusion was made in *Herring et al.* [1991] although the uncertainties are now about an order of magnitude less. The details of the estimates of the long period terms are shown in Figure 7. Interestingly, the signs of the differences between the individual analyses and the MHB2000 nutation series are commonly the same for the long period terms, although the magnitudes are often more than a factor of two different. The indication is that there may be significant differences to the geophysical model but the uncertainties of these differences are large.

The class of comparisons that we do not fully understand is the difference between results obtained from data sets generated by taking every second measurement from one data set and the remaining measurements for the other data set. This division we refer to as an odd/even numbered division. The two sets generated this way have no overlapping data and in that sense should be independent. Also the divided data sets are of similar duration to the original data set and therefore the long period terms can be determined with uncertainties about square root of two larger than the complete data set. Other divisions of the data based on, for example, earlier and later data do not allow the longer period terms to be well determined because of the decreased duration of the data. For the both the USNO and GSFC analyses, the odd/even numbered comparison generate results that are fully consistent with random noise in the measurements. The GSFC analysis is almost too good, hovering near the 5% confidence interval for the whole range of periods. The differences in the estimates of the amplitudes of the long period nutations from the odd/even data distributions are fully consistent with random white noise. The most logical explanation for this type of result (given that the differences between analysis center results shows that the noise spectrum is not completely white) is long period correlations in the measurement errors. However, the radio telescopes used in VLBI measurements typically change dramatically between experiments adjacent in time, so it would seem unlikely that the correlations arise from processes occurring at radio telescopes themselves. The common thread between alternating VLBI experiments could be the radio sources used although even here exactly the same radio sources are not likely to be used in alternating experiments. However, there are lists of radio sources that are considered good for VLBI measurements and so each experiment is scheduled using a group of common radio sources. If the correlations arise from common radio sources, the expectation would then be that the USNO and GSFC analyses would agree better with each other. The results in Table 2 show that the differences between the two analyses are quite large in terms of the standard deviations of the measurements.

The most likely explanation of the longer period correlations common within one analysis but not between different analyses is subtle effects of differences in the modeling parameters of the two groups. While the two groups use the same analysis software, there are differences in the values of the parameters used in the some of the models. For example, the diurnal and semidiurnal Earth rotations models used are slightly different although this is not the likely origin because the effects of these models is largest for the higher frequency nutations. More detailed analysis of the effects of the model differences between the analysis centers would seem warranted. However, this will be difficult because for a single experiment, the differences will be small. Based on the differences between the estimates of nutation amplitudes for the GSFC and USNO analyses, the difference in the nutation angle estimates on a single experiment will be <50 μas which is small compared to the RMS difference between these two groups of about 160 μas.

The final class of evaluations we have made is to compare the MHB2000 nutation series with nutation angle measurements that do not overlap in time with those used in its derivation. These new measurements are from experiments conducted at the beginning of 2000. The recent results from the analysis of the year 2000 VLBI experiments (data available from the IVS) are shown in Figure 8 along with the predicted variations from the MHB2000 nutation series. For the amplitude of the freely excited RFCN mode we used the values at the end of 1999. We also tested estimating the value of RFCN free mode from the 6-months of data in 2000 but the uncertainties were sufficiently large (approximately 100 μas) that a change in value could not be definitely concluded. The results do suggest that the value has continued to increase in accord with the 1998-2000 variation from the “series” analysis results shown in Figure 5. For the GSFC, USNO and IAA analyses, the differences between new measurements and the predictions have WRMS scatters, for the combined $\Delta\psi \sin\epsilon_0$ and $\Delta\epsilon$ residuals, of 88, 129, and 121 μas and χ^2/f of 0.42, 0.65, and 0.93 (after applying the error model in Equation 1). For the number of degrees of freedom in the calculations, none of these values differ significantly from unity at the 95% confidence interval. Within the statistical framework of this paper, the differences between the new measurements and the predictions from the MHB2000 nutation series are consistent with random error.

From the error analysis we conclude that the standard deviations for the nutation amplitudes in Table 1 with periods less than 400 days are realistic. They have been scaled by a factor of 2 to account for the double use of the VLBI data and that the data division tests can only verify the quality of the results with a variance of twice that of the complete data set. The realistic uncertainty of the long period terms is more difficult to assess although the algorithm used for the short period terms is likely to be too optimistic. Use the χ^2/f of the 12 amplitudes of the terms with periods between 1000 and 1600 days as an indicator of the quality, suggest that the uncertainties of the long period terms need to be multiplied by a further factor of two. Such a multiplier increases the uncertainty of the amplitude of the 18.6-year nutation to 38 mas resulting in a χ^2/f of the four amplitude differences with 18.6-year period of 1.7. This value of χ^2/f is not significantly different at a 95% significance level from the random noise expectation with 4-degrees of freedom.

Conclusions

The analysis of over 20-years of VLBI data yields estimates of the nutation amplitudes with standard deviations of 5 μas for the nutations with periods less than 400 days. At this level of uncertainty, the estimated amplitudes are consistent with geophysically based MHB2000 nutation series. For periods longer than 400 days, the estimated amplitudes deviate from MHB2000 by up to 56 μas. Analysis of the errors in these estimates suggests that the uncertainty of the longest period terms (18.6-year period) is approximately 38 μas. There is some indication that the deviations of the long period terms may be significant but with the current duration data sets any conclusion of deviation is tenuous. Although we have analyzed a long series of data, additional data added at this time will help resolve the long period terms. The early part of the 20-year data set is of much poorer quality than later data. In particular, there is a dramatic improvement in the quality of regularly spaced measurements when the International Radio Interferometric Surveying (IRIS) program started in 1984. By 2003, there will be more than 18 years of this higher quality data and we should expect a dramatic improvement in the quality of the estimates of the long period terms. If the data quality were uniform over the 19-year interval, and only white noise were present, the estimates 18.6-year nutation amplitude would have the same standard deviation as the short period terms. Currently, there is about a factor of four difference in the standard deviations of the short and long period terms. The features of the MHB2000 nutation model needed to explain the VLBI data are discussed in Mathews et al. [2001].

The time variable free excitation of the RFCN nutational mode is likely to be the process that ultimately limits our ability to make geophysical inferences about the Earth from nutational studies. The amplitude of the RFCN free mode has changed from about 300 μas to almost zero over the last 20 years and now seems to be increasing again. The precise excitation mechanism for this mode is not known but earlier studies indicate that atmospheric pressure variations are a prime candidate. If this is the mechanism then the atmospheric angular momentum data sets, produced mainly to study polar motion and LOD variations, could also be used to determine the free excitation of the RFCN. This type of comparison would be useful for assessing the quality of the AAM data sets at these high frequencies. In turn, such comparisons will also yield a better understanding of why there is a loss of coherence between geodetically determined polar motion and LOD excitation and AAM inferred excitations for periods less that a week. Currently, it is not clear how much of the coherence loss is due to noise in the geodetic measurements, noise in the AAM data sets, and the role of other excitation sources such as the oceans. Irrespective of the excitation source, it is clear that for precise astrometric observations and the continued development of geophysical models based on nutation data, continued monitoring of the free RFCN mode will be needed.

Acknowledgments

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Figures:

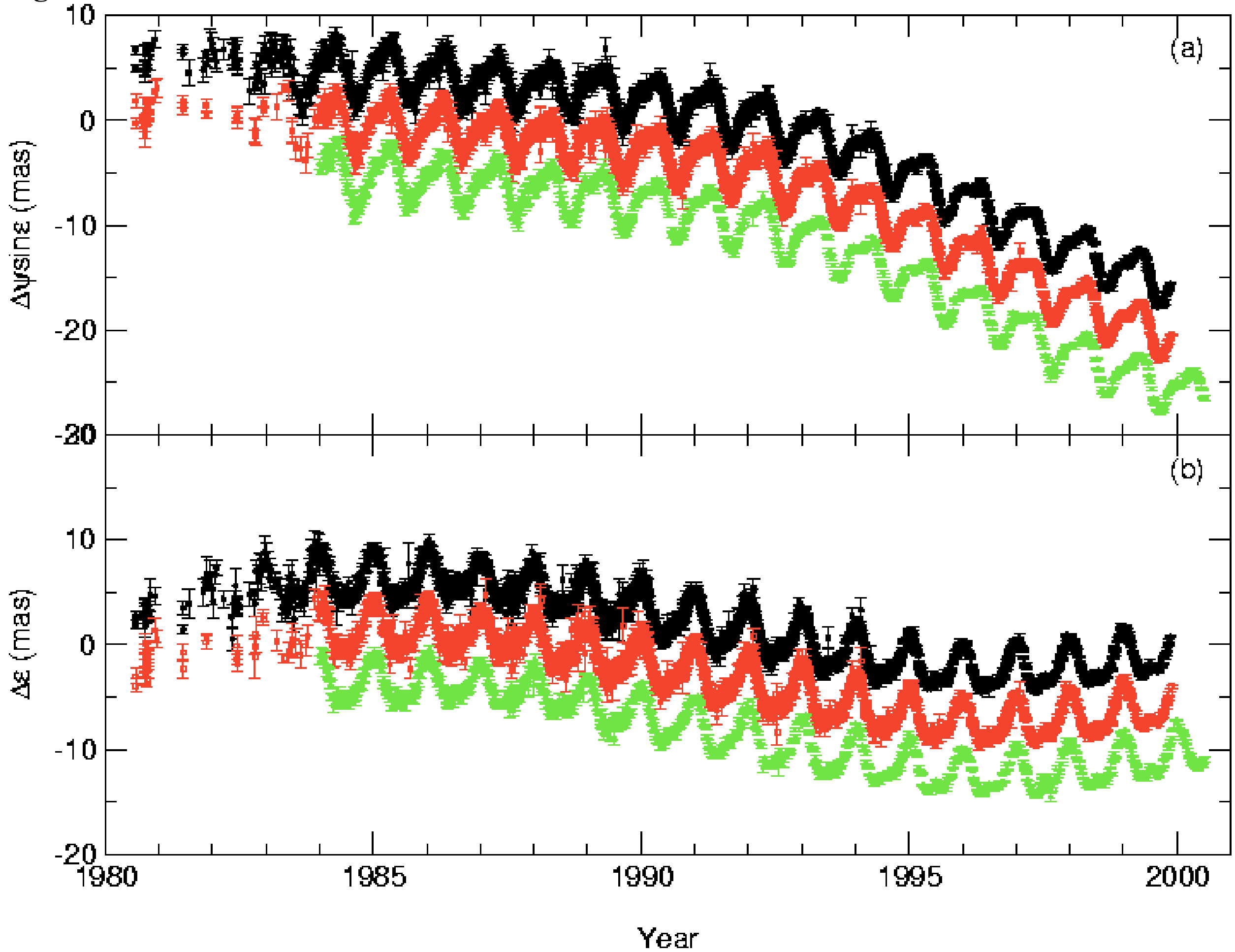

Figure 1. Nutation angle data sets used in this paper shown as differences between the VLBI measured nutations (a) $\Delta\psi \sin \varepsilon_0$ and (b) $\Delta\varepsilon$ and the IAU-1980 nutation series. Three sets of data are shown: closed circles (top), GSFC analysis; open squares (middle), USNO analysis; and open triangles (bottom), IAA analysis. For clarity the data sets have been offset from each other.

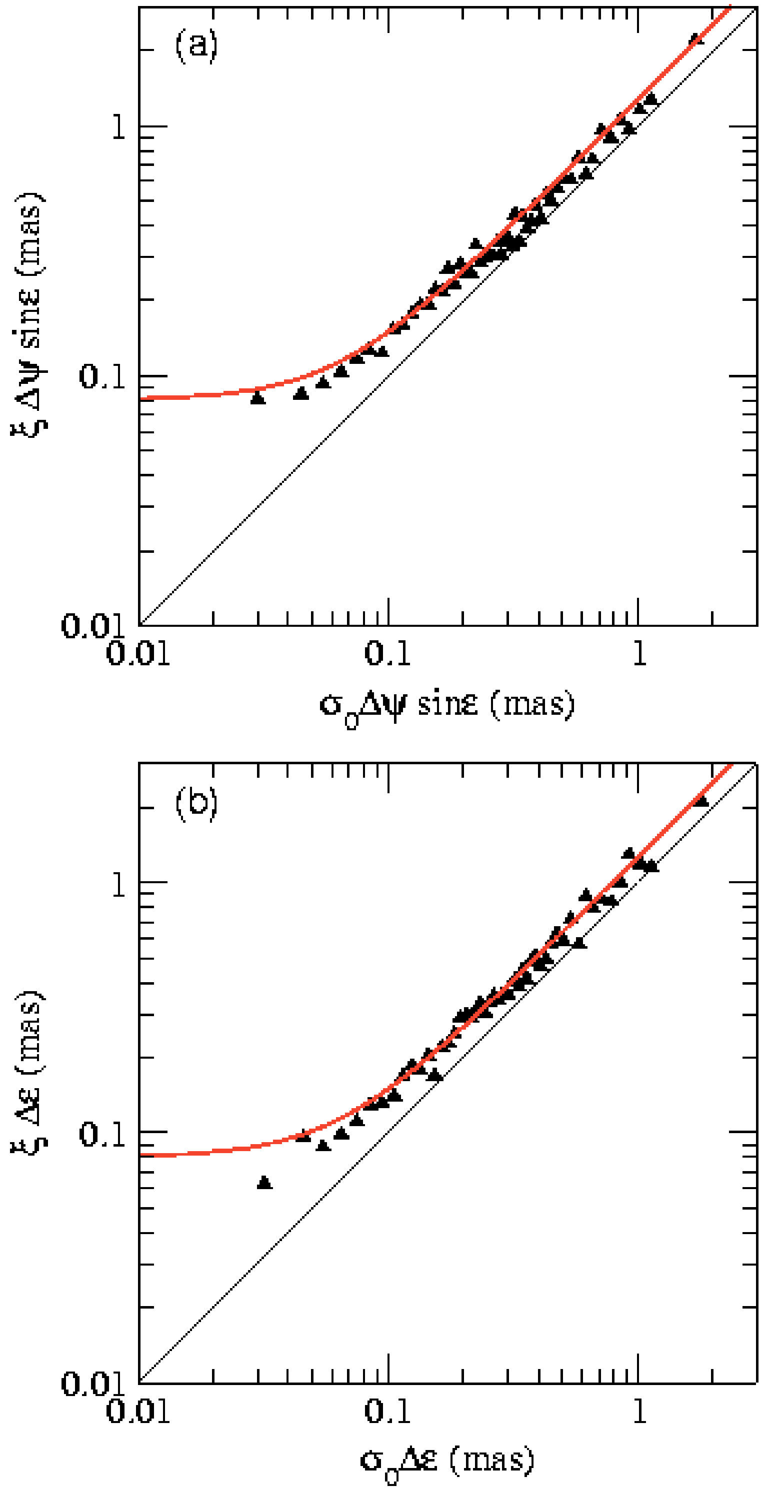

Figure 2. Comparison of weighted-root-mean-square (WRMS) scatter of nutation angle residuals for (a) $\Delta\psi$ sin ε_0 and (b) $\Delta\varepsilon$ and the expected scatter based on the VLBI estimates of the standard deviations. The closed triangles show the values from the data analysis and the solid dark line is the model given in equation (1) with parameters σ_o^2=(0.08 mas)^2 and ξ=1.6. The light solid shows the expected relationship if the WRMS scatter matched the expected standard deviations.

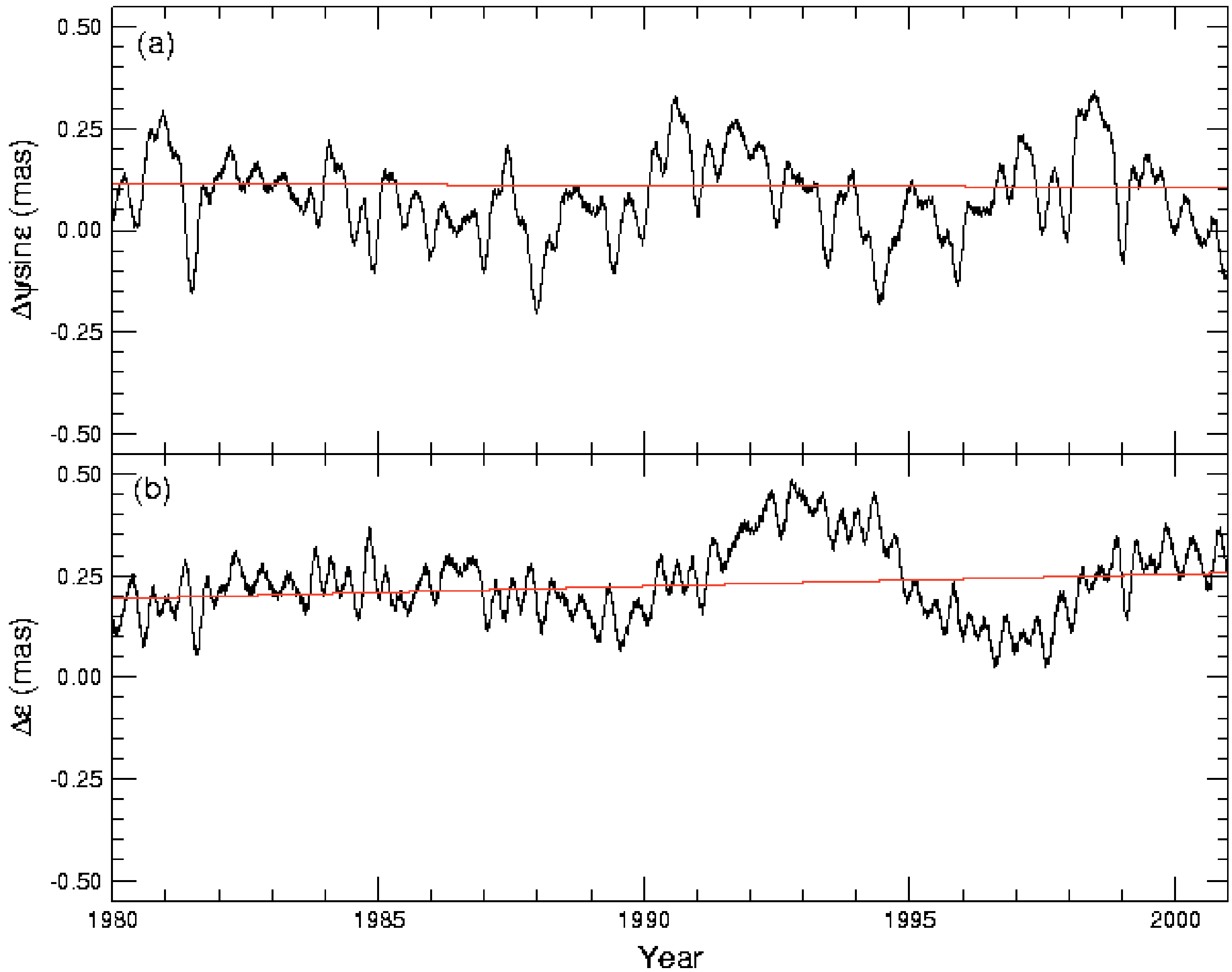

Figure 3. Contributions of planetary terms in the REN-2000 nutation series to (a) $\Delta\psi \sin\epsilon_0$ and (b) $\Delta\epsilon$. The mean and RMS scatter of the planetary contribution between 1980 and 2000 are 0.08 mas and 0.10 mas for $\Delta\psi \sin\epsilon_0$, and 0.24 mas and 0.09mas for $\Delta\epsilon$. The large mean value for $\Delta\epsilon$ is part of a long period variation in $\Delta\epsilon$ which appears to arise mainly from two terms with a –2:5 resonance between Jupiter and Saturn that generate nutations with periods near 854 years and amplitudes of 0.19 and 0.40 mas. The contributions from these two terms only are shown with the light almost straight lines in the Figure.

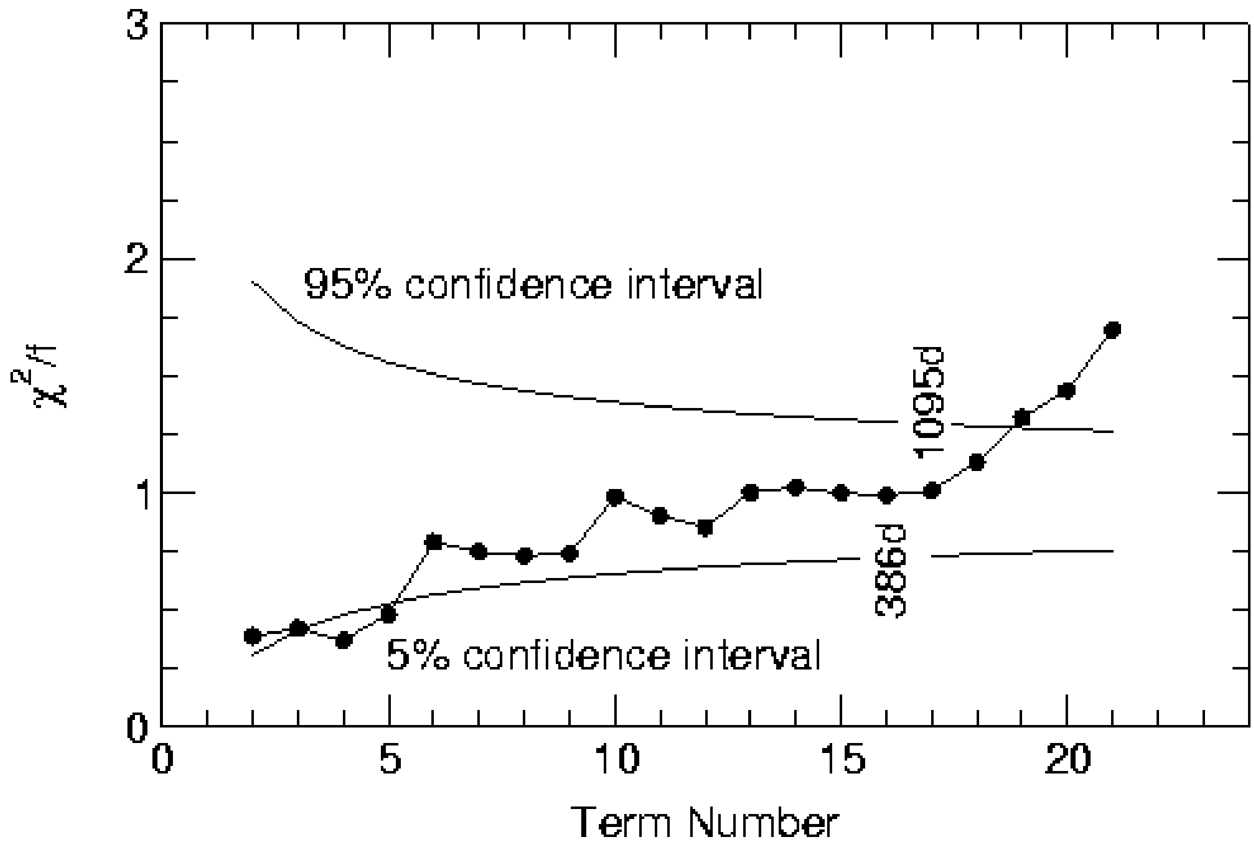

Figure 4. Statistics of the differences between the estimated amplitudes of the nutation series terms and those in the MHB2000 nutation series. Results are shown as χ^2/f versus the maximum period of the coefficients used in computing the statistics. The maximum period is expressed as a term number where the number is based on the decreasing periods in Table 1. The four amplitudes at each period (prograde, retrograde, and in- and out-of-phase) are counted as one term. The 95% and 5% confidence intervals are shown based on the number differences included in each χ^2 calculation. Term number 16, corresponding to periods less than or equal 386 days, and term number 17, periods less than or equal to 1096 days, are marked on the plot. If taken as a group, the long period terms have a χ^2/f of 3.92 for *f*=20 with a WRMS scatter of 16 μas.

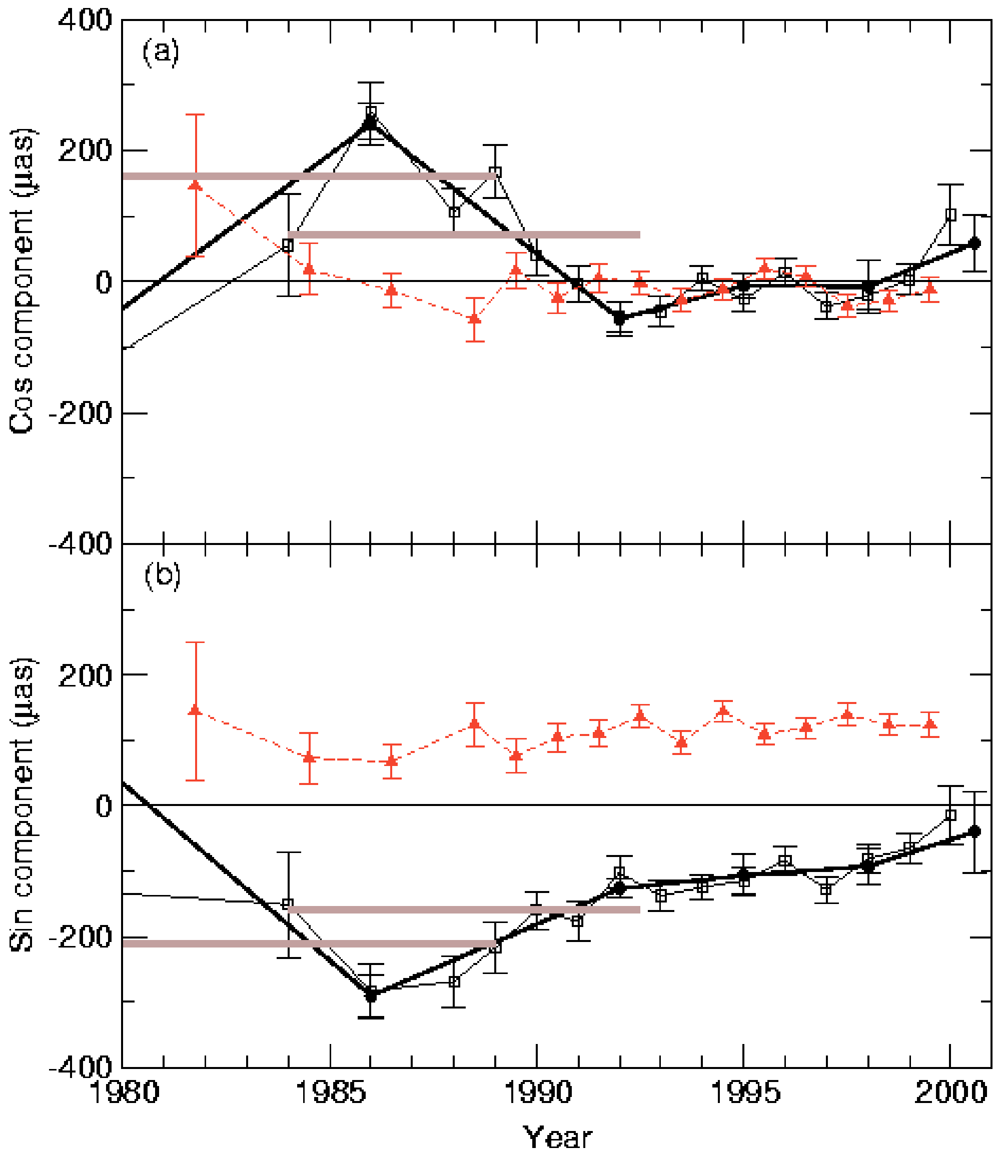

Figure 5. Time dependent estimates of the (a) cosine and (b) sine components of the freely excited RFCN mode and the prograde annual nutation. The closed circles with thick solid lines are from the "amplitude" analysis in which corrections to the amplitudes of the 88 largest nutation terms are estimated in addition to the time dependent RFCN variations. The open squares with solid lines are from the "series" analysis in which only the time dependent RFCN terms are estimated i.e., the coefficients of the MHB2000 nutation series are not estimated except for the prograde annual. The triangles with dotted lines are the piece-wise constant estimates of the time dependent differences between prograde annual nutation and the MHB2000 value without accounting for the S1-thermal tide. The points are shown at the center of the regions over which the value is constant. The stippled horizontal lines are the estimates for the RFCN free mode from *Herring et al.* [1991] and *Herring and Dong* [1994]. The horizontal length of the line shows the interval of data used in each analysis. The estimates for the RFCN mode before 1980 are not shown due to their large uncertainty. For the amplitude analysis, they are (62+111*i*)±160 µas, and for the series analysis (-21-159*i*)±255 µas where we complex notation to denote cosine and sine terms.

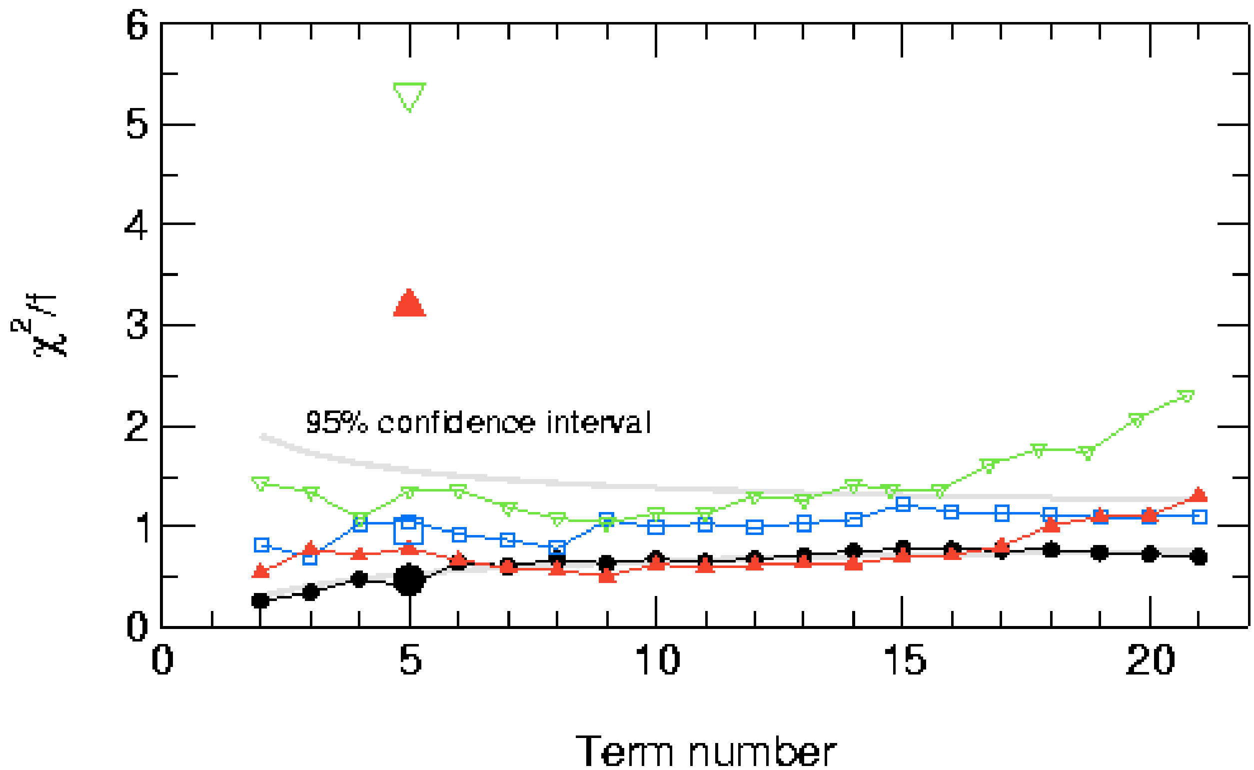

Figure 6. Statistics of differences between the estimated amplitudes of the nutation series terms from different analysis types shown as function of the maximum period of the coefficients used in computing the statistics. In part (a), results are shown as χ^2/f, computed with the sum of the variances from the pairs of analyses that use independent data or with the variances of one analysis when the data sets and/or analyses are correlated. In part (b), the WRMS scatters of the differences in amplitude in μas are shown. The maximum period is expressed as a term number where the number is based on the decreasing periods in Table 1. The four amplitudes at each period (prograde, retrograde, and in- and out-of-phase) are counted as one term. The 95% and 5% confidence intervals are shown based on the number differences included in each χ^2 calculation. For the points connected by a solid line, the filled-solid circles are for the GSFC "odd-numbered" minus "even-numbered" experiments (variances summed); the filled triangles are for the same type of analysis using the USNO data set (variances summed); the open squares are for the USNO analysis minus the GSFC analysis (USNO variances); the inverted, open triangles are for the IAA analysis minus the combined GSFC and USNO analyses (IAA variances). The large symbols, not connected to any line, follow the same convention and are for the twenty amplitudes with periods greater than 1000-days. They are shown at an equivalent term number such that the confidence interval has the correct value for the number of degrees of freedom. The variances used here are derived from the data re-weighting only (Equation 1) and have no additional factors applied.

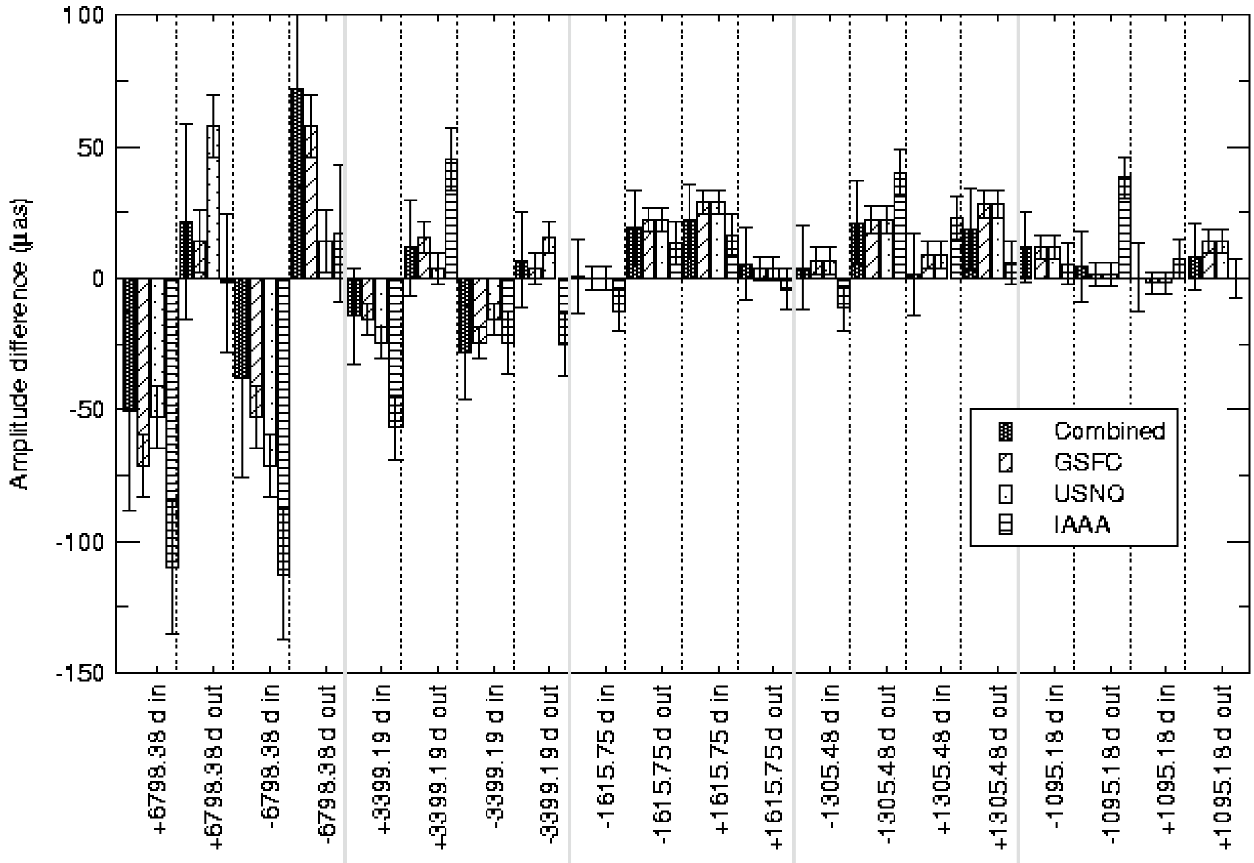

Figure 7. Differences between the long-period nutation amplitude estimates for different data sets and the MHB2000 nutation series values. The legend in the figure shows the analyses used with "combined" meaning the combination of the GSFC and USNO analyses. The WRMS differences and χ^2/f for the analyses are: Combined 20 µas, 1.6; GSFC 23 µas, 18.6; USNO 18 µas, 8.0; and IAA 32 µas, 11.9. The χ^2/f for the combined analysis is computed using the uncertainties given in Table 1 (i.e., the scaling factors have been applied). For the other analyses, the χ^2/f is computed using the standard deviations computed based only but changing the nutation angle standard deviations in accord with Equation (1).

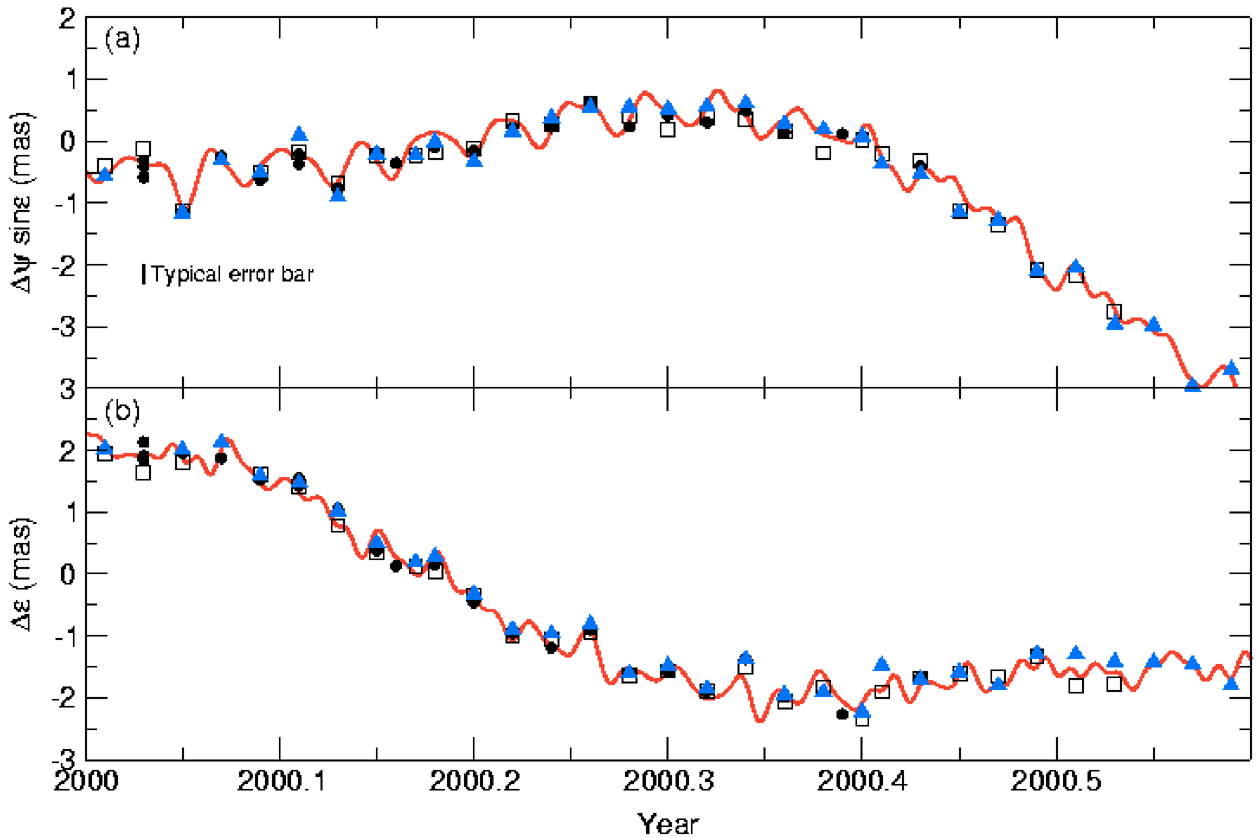

Figure 8. Comparison of recent nutation angle determinations with the predicted part of the MHB2000 nutation series. The solid line is the MHB2000 nutation series with the mean removed and with the components of the RFCN free mode kept at the values from the end of 2000. The filled circles are recent GSFC results, the open squares recent USNO results, and the closed triangles recent IAA results. The WRMS differences and χ^2/f (using standard deviations computed from Equation (1)) between the measurements and MHB2000 are for (a) $\Delta\psi \sin\varepsilon$, and (b) $\Delta\varepsilon$: GSFC 84 μas, 0.37, and 91 μas, 0.47 (23 values); USNO 130 μas, 0.65, and 128 μas, 0.66 (27 values); and IAA 129 μas, 1.04 and 113 μas, 0.82 (32 values). An average error bar is shown in part (a) of the figure. The error bars vary between 0.10 and 0.25 mas.

Table 1. Estimated complex amplitudes of the nutation series from the VLBI analysis. The residuals are the differences to the MHB2000 nutation series. The uncertainties shown are twice the values obtained from the analysis of the combined GSFC and USNO data sets, with data standard deviations computed using Equation (1), for the terms with periods less than 400-days, and four-times these standard deviations for periods longer than 400-days (see Error Analysis section for discussion of uncertainties).

Term Number	Period (days)	In-phase (mas)	Residual (µas)	± (µas)	Out-of-phase (mas)	Residual (µas)	± (µas)
21	-6798.38	-8024.825	-50	38	1.454	21	37
	6798.38	-1180.496	-37	38	-0.033	71	37
20	-3399.19	86.121	-14	18	-0.017	11	18
	3399.19	3.586	-28	18	0.008	7	18
19	-1615.75	-0.004	0	14	0.019	19	14
	1615.75	-0.105	21	13	0.005	5	13
18	-1305.48	0.303	3	16	0.021	20	16
	1305.48	2.126	1	15	0.020	18	15
17	-1095.18	0.226	11	13	0.004	4	13
	1095.18	-0.224	0	12	0.008	8	12
16	-386.00†	-0.158	-9	23	0.004	-	-
	386.00	-0.709	-1	6	-0.010	-9	6
15	-365.26	-33.039	7	11	0.339	7	13
	365.26*	25.645	-1	7	0.131	-9	7
14	-346.64	-0.565	7	8	-0.003	-5	8
	346.64	-0.063	6	6	-0.012	-11	6
13	-182.62	-24.568	-5	5	-0.059	-15	5
	182.62	-548.471	-0	5	-0.499	2	5
12	-121.75	-0.941	-0	5	-0.002	-0	5
	121.75	-21.502	-3	5	-0.015	4	5
11	-31.81	-3.059	1	5	-0.008	-1	5
	31.81	3.185	0	5	0.003	2	5
10	-27.55	-13.798	8	5	-0.050	-15	5
	27.55	14.484	2	5	-0.002	-3	5
9	-23.94	0.046	-2	5	-0.007	-7	5
	23.94	1.189	3	5	-0.004	-3	5
8	-14.77	-1.200	-0	5	-0.012	-7	5
	14.77	1.324	2	5	-0.003	-1	5
7	-13.78	-0.545	6	5	0.000	2	5
	13.78	0.613	0	5	-0.002	-1	5
6	-13.66	-3.639	8	5	-0.025	-11	5
	13.66	-94.196	2	5	0.120	-4	5
5	-9.56	-0.085	5	5	0.001	1	5
	9.56	-2.464	-1	5	0.014	7	5
4	-9.13	-0.452	3	5	-0.005	-3	5
	9.13	-12.449	-1	5	0.035	0	5

3	-9.12	-0.289	3	5	-0.007	-5	5
	9.12	-2.346	-1	5	0.011	4	5
2	-7.10	-0.054	1	5	-0.002	-1	5
	7.10	-1.593	-2	5	0.003	-3	6
1	-6.86	-0.040	4	5	-0.001	-0	5
	6.86	-1.280	-0	5	-0.002	-7	5
	Secular ‡	-2.960	37	30	-0.237		12

† This term is strongly effected by the piecewise linear estimation of the complex amplitude of the freely excited RFCN mode. The out-of-phase 386-day period amplitude has been constrained to the MHB2000 apriori value (see more discussion in data analysis section).
‡ The secular terms are the linear rates of changes of □□ and □□(mas/yr). The residuals and standard deviations are in micro-arc-sec per year (□as/yr). The residual to *d□□/dt* is from the MHB2000 estimate of the correction to the precession constant.
* MHB200 includes a correction of -9 □as, in-phase, and 118 □as, out-of-phase, to the prograde annual nutation to account for effects most likely arising from the thermal S1 atmosphere tide.

Table 2. Statistics of the nutation angle residuals from different analyses and the differences between the nutation angles from the different analysis centers (see text for descriptions and discussion).

Data	#	Series		Amplitude		MHB2000†		GSFC		UNSO	
		μas*	χ^2/f	μas*	χ^2/f	μas*	χ^2/f	μas*	χ^2/f	μas*	χ^2/f
$\Delta\psi \sin\varepsilon_0$											
All	5687	186	0.952	183	0.915						
GSFC	2974	178	1.040	174	0.995	179	1.047				
USNO	2713	200	0.849	197	0.828	202	0.859	158	0.544		
IAA	981	182	1.264	176	1.181	188	1.323	181	1.272	225	1.225
$\Delta\varepsilon$											
All	5687	193	0.996	189	0.949						
GSFC	2974	186	1.105	180	1.036	186	1.102				
USNO	2713	204	0.865	202	0.842	205	0.868	169	0.615		
IAA	981	188	1.339	178	1.203	190	1.364	197	1.504	228	1.259

* Weighted-root-mean-square (WRMS) scatter of the residuals or the differences. χ^2/f is computed after modifying the nutation angle standard deviations according to Equation (1).
† MHB2000 uses the temporal changes in the free RFCN mode determined from the combined GSFC and USNO data sets.