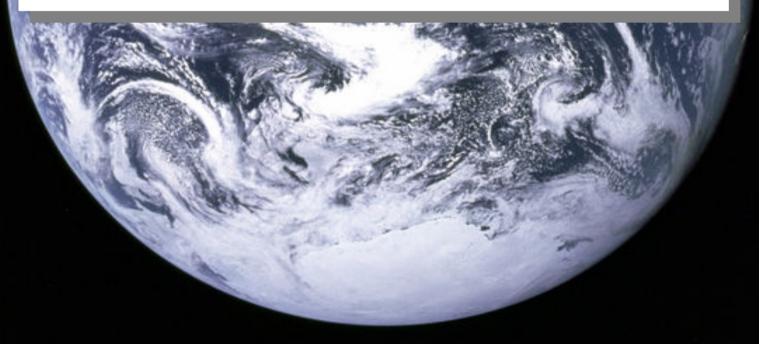
# **Environmental Geodesy**

#### Hans-Peter Plag Nevada Bureau of Mines and Geology and Seismological Laboratory, University of Nevada, Reno, NV, USA, hpplag@unr.edu.



# **Environmental Geodesy**

Lecture 2 (January 25, 2011): Geodetic Reference Systems and Frames

- Geodetic Reference Systems
- Celestial Reference Systems and Frames
- Terrestrial Reference Systems and Frames
- Realization of ITRF through ITRS
- Relation between ITRS and other Systems
- Dynamical Reference Frames

*Measuring position and motion requires a reference system.* 

Position and motion depends on the reference system

#### Ideal World:

Reference systems can be defined by:

- three coordinate axes,
- the origin, and
- a scale

Axes either being fixed in space or having a known movement with respect to something else that is fixed.

Real world: Definition of reference systems is far more complex.

*Measuring position and motion requires a reference system.* 

Position and motion depends on the reference system.

*In geodesy, two reference systems are intrinsic:* 

- Celestial Reference System (CRS)
- Terrestrial Reference System (TRS)

and these are connected through Earth Rotation.

It is of obvious practical advantage to agree upon one definition for these systems:

- Conventional Celestial Reference System (CCRS)

- Conventional Terrestrial Reference System (CTRS)

- **Convention:** In the context of geodetic reference systems, convention refers to an agreement between groups, especially an international agreement, that is slightly less formal than a treaty.
- Examples: the agreed-upon way to transform from inertial frame to terrestrial frame, splitting three unique angles that connect the two frames to a set of conventionally defined sub-group of angles (polar motion, Earth rotation, nutation and precession).
- In geodesy, conventions often regulate ways to process data in order to ensure comparability of the resulting products.
- In many cases, standards adopted by e.g. IAG/IUGG become part of conventions.

**Standard:** In geodesy, a standard refers to an authorized model (normally authorized by IAG or IUGG or other international bodies recognized by IAG/IUGG) used to define a unit of measurement. Examples of standards are the definition of the meter, the speed of light, and similar physical constants.

#### How to gain access to a reference system?

Modern conventional celestial and terrestrial reference systems are realized through coordinates of a set of points and objects determined from observations analyzed with appropriate mathematical and physical models.

Such a realization of a reference system is denoted as reference frame.

In practice, the realization of a reference system through such a frame requires continuous monitoring of the points or objects.

Any realization also requires the specification of additional boundary conditions that the reference frame should fulfill.

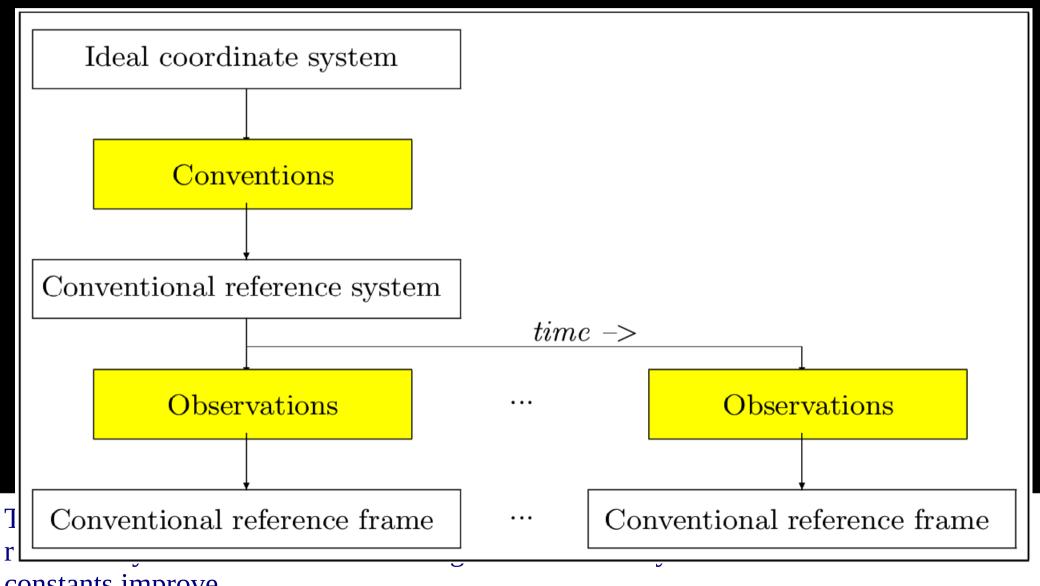
Moreover, models used to analyze the observations and to correct for disturbances in the coordinates of the points and objects are an integral part of the realization, and therefore have to be included in the convention specifying the reference system and its realization through a frame.

#### How to gain access to a reference system?

It is not always clear whether the boundary conditions and models are considered as part of the conventional reference system, part of the reference frame realizing the system or the subject of an additional convention.

There is a trade-off between the completeness of the conventions specifying the reference system and the need to change the reference system when models or constants improve.

#### How to gain access to a reference system?



constants improve.

Today, the global reference systems agreed upon in scientific communities and increasingly used for societal applications are:

- International Celestial Reference System (ICRS)
- International Terrestrial Reference System (ITRS)

and these are connected through the Earth Rotation.

*These two reference systems are realized through reference frames:* 

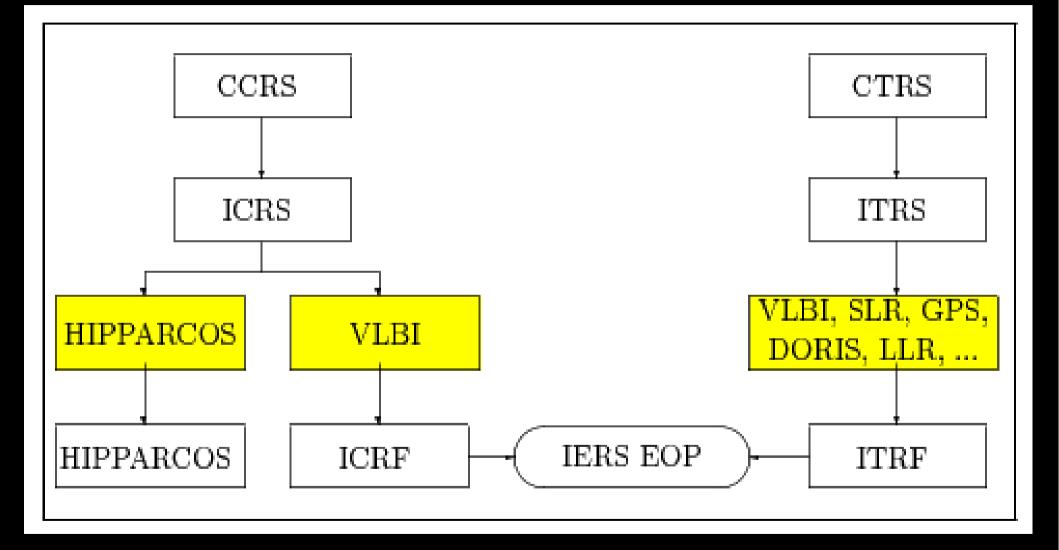
- International Celestial Reference Frame (ICRF)
- International Terrestrial Reference Frame (ITRF)

and the connections is provided by the

- Earth Orientation Parameters (EOPs).

*The Reference Systems are maintained by the International Earth Rotation and Reference Systems Service (IERS).* 

The Reference Frames and the EOPs are determined based on observations provided by the Global Geodetic Observing System (GGOS).



In addition to ICRF and ITRF, we also need a Conventional Dynamic Reference Frame (CDRF) that provides the planetary and lunar ephemerides in the selected ICRF.

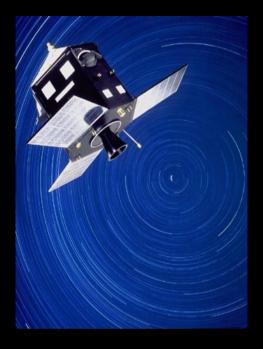
Such a frame will be derived from preferably all relevant observational data.

The adjustment of the observations requires a number of astronomical constants as well as the planetary masses, and these numbers are considered an integral part of the CDRF.

#### Before introduction of ICRF:

- Sequence of six astrometric Catalogues of Fundamental Stars providing high precision positional data for a selection of stars:
- *FK4*: *published in 1963*; *1535 stars in various equinoxes from 1950.0 to 1975.0*.
- FK4S: amendment to FK4 containing further 1987 stars.
- FK5: update of FK4 in 1988; new positions for the 1535 stars.
- FK5 Extension: addition of 3117 new fundamental stars, published in 1991.
- FK6: 2000 update of FK5 correlated with the ICRF through the HIPPARCOS satellite.
- FK6 has two parts, FK6(I) and FK6(III) containing respectively 878 and 3272 stars.

The Reference Systems are maintained by the International Earth Rotation and Reference Systems Service (IERS).



#### Hipparcos:

- ESA space astrometry mission;
- pinpointed the positions of more than one hundred thousand stars with high precision, and more than one million stars with lesser precision.
- Launch: in August 1989
- Operations ceased in March 1993 (3.5 years)
- Main instrument: Hipparcos Catalogue of 118,218 stars with the highest precision.
- Auxiliary star mapper: lesser but still unprecedented accuracy;
- Tycho Catalogue of 1,058,332 stars.
- Tycho 2 Catalogue, completed in 2000: 2,539,913 stars.

*FK5* reference system did not fulfill the demands of modern astrometry

*Example: FK5 constant of precession is wrong by 0.3 arcsec/century.* 

Since 1987, the International Earth Rotation Service (IERS) determined Earth rotation parameters with respect to extragalactic objects using VLBI. The results were then transformed into the FK5 system, which was the "official" system.

International Astronomical Union (IAU) in 1991:

- base the realization of its CCRS on kinematic rather than a dynamical definition

- use distant extragalactic objects and to adopt directions which would be fixed with respect to a selected set of these objects.

#### New concept:

- axes fixed with respect to distant matter in the Universe;
- ensures that the reference coordinates do not rotate with respect to a large portion of the Universe surrounding our galaxy.
- the new CCRS is quasi-inertial.

IAU recommendations in 1992:

- origin of the new CCRS is to be at the barycenter of the solar system
- the axes are to be fixed with respect to the quasars.
- directions consistent with their previous realizations, that is the FK5 origin of right ascension and pole (within the uncertainties of the FK5).

The IAU resolutions explicitly introduce the Theory of General Relativity as the basis for all theoretical and data analyses related to space and time.

Ensures that the new CCRS would not be detrimental to the analysis of observations from the highly accurate astrometric techniques.

Choice of extragalactic objects to realize the fiducial directions was possible due to Very Long Baseline Interferometry (VLBI).

Already in 1991, it was clear that a realization of the CCRS would become available soon for radio wavelength.

However, also in 1991, the IAU decided that such a reference system will not become the actual reference frame for astronomy until it would be completed by a catalogue in the optical range, having in mind the HIPPARCOS Catalogue.

IERS published in 1997 the International Celestial Reference Frame (ICRF)

- *The ICRF includes the positions of 606 extragalactic radio-sources:*
- 212 were considered to be fundamentally defining the frame.
- positions of the other sources were given in the frame, but since they are observed less, their positions are less accurate.
- positions of the 212 fundamental sources were determined better than 0.6 mas in right ascension and declination.

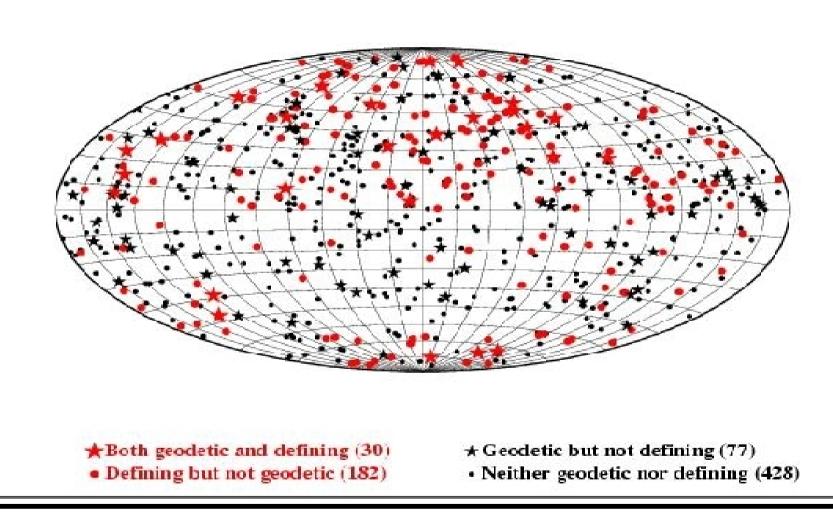
In parallel, Hipparcos stellar reference frame

- was astrometrically aligned with the ICRF to within  $\pm 0.6$  mas at the central observation epoch of Hipparcos at 1991.25 and in spin/rotation within  $\pm 0.25$  mas/year.

- provided the primary realization of the ICRS at the optical wavelengths.

IAU accepted in 1997 the ICRS as the new CCRS and the ICRF and Hipparcos as its realization in the radio and optical wavelengths, respectively.

#### **ICRF-Ext.2 Sources**



Source locations of ICRF-Ext.2. The second extension of ICRF was computed based on VLBI data obtained between mid-1995 and the end of 2002 May and included an additional 109 new sources.

Attempts to define and realize reference systems or at least global reference surfaces and parameters of the Earth were made in the beginning of the 19th century, though mainly on national or regional level.

Examples:

- sequence of reference ellipsoids starting with the one defined by Bessel in 1841, which was used for the German national reference frame (DHDN) determined by triangulation.

- Other such national reference frames were the OSGB36 in the U.K. and NTF in France.

- After World War II, the ED50 was introduced as a European unification.

Most attempts were separate for systems for geographical coordinates and height datums.

Geoidal reference surfaces provide another example for a reference, which, in principle, could have been used as a global reference for height.

Name of ellipsoid	semimajor	flattening	applied for
	<b>axis</b> <i>a</i> [m]	f = (a - b)/a	
Geodetic Reference System 1980 (GRS80)	6 378 137.	1:298.25722	World Geodetic
			System 1984
World Geodetic System 1972 (WGS72)	6 378 135.	1:298.26	World Geodetic
			System 1972
Geodetic Reference System 1967	6 378 160.	1:298.25	Australian Datum
			1966
			South American
			Datum 1969
Krassovski (1942)	6 378 245.	1:298.3	Pulkovo Datum
			1942
International (Hayford 1924)	6 378 388.	1:297.0	European Datum
			1950
Clark (1866)	6 378 206.	1:294.98	North American
			Datum 1927
Bessel (1841)	6 377 397.	1:299.15	German DHDN

List of reference ellipsoids introduced in the past.

One of the first application of a global 3-D reference frame: Earth rotation observations starting in middle of 19th century.

Subsequently, a number of global reference systems were introduced on national or international level and realized in various ways.

A prominent role was attached to the international scientific bodies responsible for Earth rotation monitoring, such as the International Latitude Service (ILS), the International Polar Motion Service (IPMS), the Bureau International de l'Heure (BIH), and, currently, the IERS.

The IAG has a historical role in the development of conventional reference frame responding to or even prompting the development of observational techniques.

*The IAG Commission RETRIG continued to update ED50 until 1987 through ED79 and ED87.* 

In 1967, the IAG and also the IAU adopted the Conventional International Origin (CIO) frame, with a pole defined for the epoch 1903.0 as the mean of the ILS observations of the pole during the period 1900.0 to 1906.0.

IAU and IAG are the two scientific bodies which also introduced the systems presently used for most accurate applications.

Until 1984: international accepted TRS was the CIO-BIH system, which was realized by use of Earth Rotation Parameters (EOP).

The frame was a network of astronomical instruments with coordinates determined by astronomical observations.

In 1984, the BIH started to produce the BTS, which was realized through a new type of TRF based on space geodesy.

In 1987, the IERS was established by IUGG and IAU as a FAGS services with the mission to materialize the a CRS and a TRS as well as determine EOP. The IERS replaced the BIH.

In 1979, the IUGG accepted the Geodetic Reference System 1980 (GSR80), which, among others, specifies relevant constants and the geometrical and physical parameters of the figure of the Earth.

parameter	symbol	value			
defining constants					
equatorial radius of the Earth	a	6378137 m			
geocentric gravitational constant	GM	$3986005 \cdot 10^8 \text{ m}^3 \text{s}^{-2}$			
(including the atmosphere)					
dynamical form factor	$J_2$	$108263 \cdot 10^{-8}$			
(excluding permanent tides)					
angular velocity of the Earth	$\omega$	$7292115 \cdot 10^{-11} \text{ rad s}^{-1}$			
derived geometrical parameters					
semiminor axis (polar radius)	b	6356752.3141 m			
first excentricity	$e^2$	0.00669438002290			
fattening	f 1: 298.257222101				
mean radius	$R_1$	6371008.7714 m			
radius of sphere with same surface	$R_2$	6371007.1810 m			
radius of sphere with same volume	$R_3$	6371000.7900 m			
derived physical parameters					
normal potential at ellipsoid	$U_0$	62636860.850 m <sup>2</sup> s <sup>-2</sup>			
Normal gravity at equator	$g_{ m e}$	$9.7803267715 \text{ m s}^{-2}$			
Normal gravity at pole	$g_{ m p}$	9.8321863685 m s <sup>-2</sup>			

The development of particularly VLBI and SLR led to a number of single or two-technique defined reference frames used mainly for scientific geodetic purposes.

IERS started to produce a sequence of annual realizations of the IERS Terrestrial Reference System (the successor of the BTS) with the IERS Terrestrial Reference Frame 1988 (ITRF88).

1991 IUGG Assembly in Vienna: Resolution 2 lay ground for a formal acceptance of this TRF through the relevant international scientific organizations as the CTRS.

The system specified by this Resolution was at that time already under implementation by the IERS through ITRF88, ITRF89, ITRF90.

Today, the most accurate global terrestrial reference system is maintained by the IERS through international cooperation. The ITRS is specified in detailed in the IERS Conventions.

- In Newtonian theory, the ideal Terrestrial Reference System can be considered to be a three-dimensional coordinate system with origin close to the Earth center and co-rotating with it.
- The geometry of an Euclidian affine space of dimension 3 provides a standard model of such a system. Using the affine frame (O,E), where O is a point in space called origin and E a vector base of the associated vector space.
- Currently, *E* is restricted to be orthogonal with all base vectors having the same length.
- The common length of the base vectors is named the scale of the TRS.
- However, it should be kept in mind that this this Newtonian model is valid to visualize the concept for practical users, but the actual definition of the CTRS today has to be based on the General Theory of Relativity, where the CTRS is a local Earth system as specified in the IAU 1991resolutions.

#### The ITRS follows the criteria:

- a) It is geocentric, the center of mass being defined for the whole Earth, including oceans and atmosphere.
- b) Its scale is that of a local Earth frame, in the meaning of a relativistic theory of gravitation.
- c) Its orientation was initially given by the BIH orientation at 1984.0.
- d) Its time evolution in orientation will create no residual global rotation with respect to the crust.

The unit of length is the SI meter.

The scale is obtained by appropriate relativistic modeling.

- The orientation is defined by adopting IERS Earth orientation parameters at a reference epoch.
- In case of dynamical observation techniques, an additional constraint in longitude is necessary to remove ill-conditioning.

- The IERS Reference Pole (IRP) and Reference Meridian (IRM) are consistent with the corresponding directions of the BTS within  $\pm 5$  mas.
- The BIH reference pole was adjusted to the CIO in 1967 and was kept stable until 1987.
- The uncertainty in the tie of the IRP with the CIO is  $\pm 30$  mas.
- The time evolution of the orientation is to be ensure by using a No-Net-Rotation (NNR) condition with respect to horizontal tectonic motion averaged over the whole Earth.
- There are several controversial conventions, including:
- implementation of the NNR condition,
- treatment of the permanent tide, which is in disagreement with the conventions in gravity and IUGG resolutions.

The ITRS is realized through a reference frame specifying a set of coordinates for a network of stations.

These coordinates are given as Cartesian equatorial coordinates triples  $x_i = (X, Y, Z)$  by preference.

The IERS Conventions suggest that if geographical coordinates are needed, the GRS80 ellipsoid should be used.

#### How are the point coordinates described?

on results provided by the different IERS analysis centers. The realization consists of lists of coordinates and velocities for a selection of IERS sites, which may be tracking stations or related ground markers. The station coordinates are expressed through

$$x_i(t) = x_i^0 + v_i^0(t - t_0) + \sum_{j=1}^k \delta x_i^j(t), \quad i = 1, 2, 3 \quad (4)$$

where  $x_i^0$  and  $v_i^0$  are the position and velocity at epoch  $t = t_0$ and  $\delta x_i^k$  are corrections due to the k-th process inducing time variable contributions to the coordinates. Such processes are, for example, solid Earth tide displacements, ocean loading, atmospheric loading, and postglacial rebound.

#### Regularized coordinates:

$$\tilde{x}_i(t) = x_i^0 + v_i^0(t - t_0),$$

Points form a secular polyhedron changing over time.

These coordinates are listed by IERS for each version of ITRF

These coordinates depend on the choices made for:

$$\Delta_i(t) = \sum_{j=1}^k \delta x_i^j(t).$$

Observational errors and technological improvements require monitoring and frequent updates /determinations of the reference frame.

#### Transformations between frames:

For different realizations of the ITRS, transformations are given to convert coordinates from one ITRF to another. The basic transformation formula is a seven parameter similarity transformation, often denoted as Helmert Transformation. This is given by

$$x'_{i} = sR_{ij}x_{j} + t_{i}, \ i = 1, 2, 3 \tag{7}$$

where  $x_i$  and  $x'_i$  are the coordinate vectors of the point in the unprimed and primed frame, respectively, and  $t_i$  is the vector describing the offset of the origin between the primed and unprimed system measured in scale units of the primed system. s is the scale change of the primed frame with respect to the unprimed frame, and  $R_{ij}$  is a rotation matrix

$$R_{ij} = R_{ik}^{(1)}(\epsilon) R_{kl}^{(2)}(\psi) R_{lj}^{(3)}(\omega).$$
(8)

#### Transformations between frames:

$$R_{ij} = R_{ik}^{(1)}(\epsilon) R_{kl}^{(2)}(\psi) R_{lj}^{(3)}(\omega).$$
(8)

Here, the  $(R_{ij}^{(n)}(\alpha))$  are rotation matrices describing a rotation around the *n*-th axis. For  $j = n \pmod{3} + 1$ ;  $k = j \pmod{3} + 1$ , we have

$$R_{nn} = 1$$

$$R_{nj} = R_{jn} = R_{nk} = R_{kn} = 0$$

$$R_{jj} = R_{kk} = \cos \alpha$$

$$R_{jk} = \sin \alpha$$

$$R_{kj} = -\sin \alpha$$
(10)

For infinitesimal rotations, (7) can be written as

$$x'_i = (1+\delta s)\tilde{R}_{ij}x_j + t_i \tag{11}$$

where  $\delta s$  is the incremental scale change and with

$$(\tilde{R}_{ij}) = \begin{pmatrix} 1 & \omega & -\psi \\ -\omega & 1 & \epsilon \\ \psi & -\epsilon & 1 \end{pmatrix}$$
(12)

where  $\epsilon, \psi$ , and  $\omega$  are given in radian. Then,

#### Transformations between frames:

$$x_i' = x_j + \delta s \tilde{R}_{ij} x_j + t_i$$

Representation of parameters in Helmert transformation:

changes of these parameters. In this case, for a given transformation parameter q valid at the epoch  $t_0$ , its value at time t is given by

$$q(t) = q(t_0) + \dot{q}(t - t_0).$$
(14)

Helmert transformation is rigid, does not allow for deformations of the two frames.

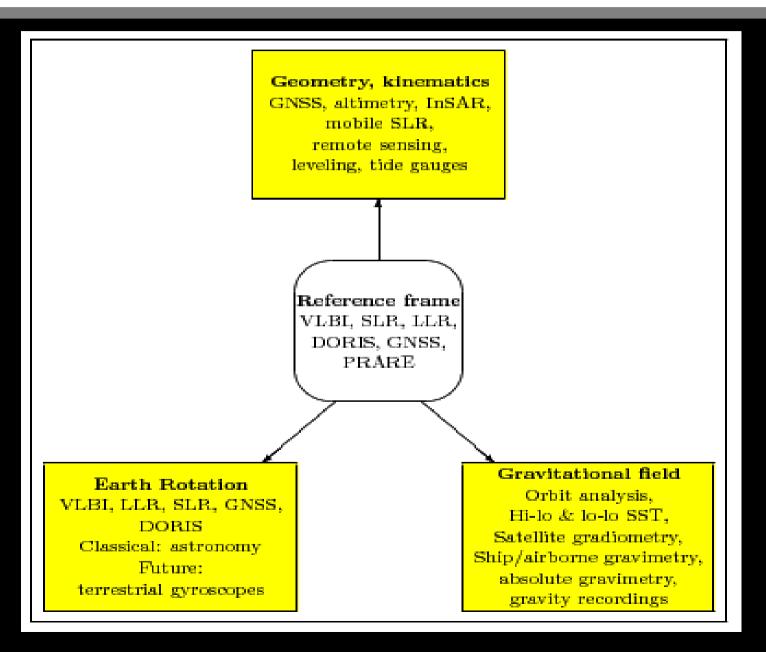
#### Comparing coordinate in the same frame:

For many applications, it is necessary to compare coordinates of a point determined at different epochs or to refer coordinates to a reference epoch different from the central epoch of observations. Within the same reference frame, this can be achieved by

$$x_i(t_r) = x_i(t_c) + (t_r - t_c) \cdot v_i$$
(15)

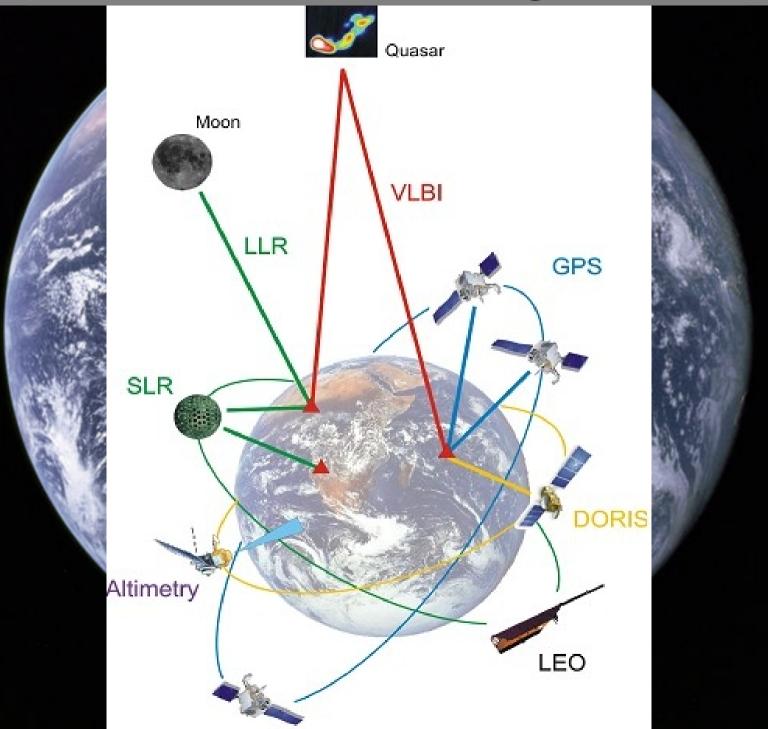
where  $t_r$  and  $t_c$  are the reference epoch and the central epoch of measurement, respectively, and  $x_i$  and  $v_i$  are the position and velocity vectors given in the relevant ITRF. If positions given

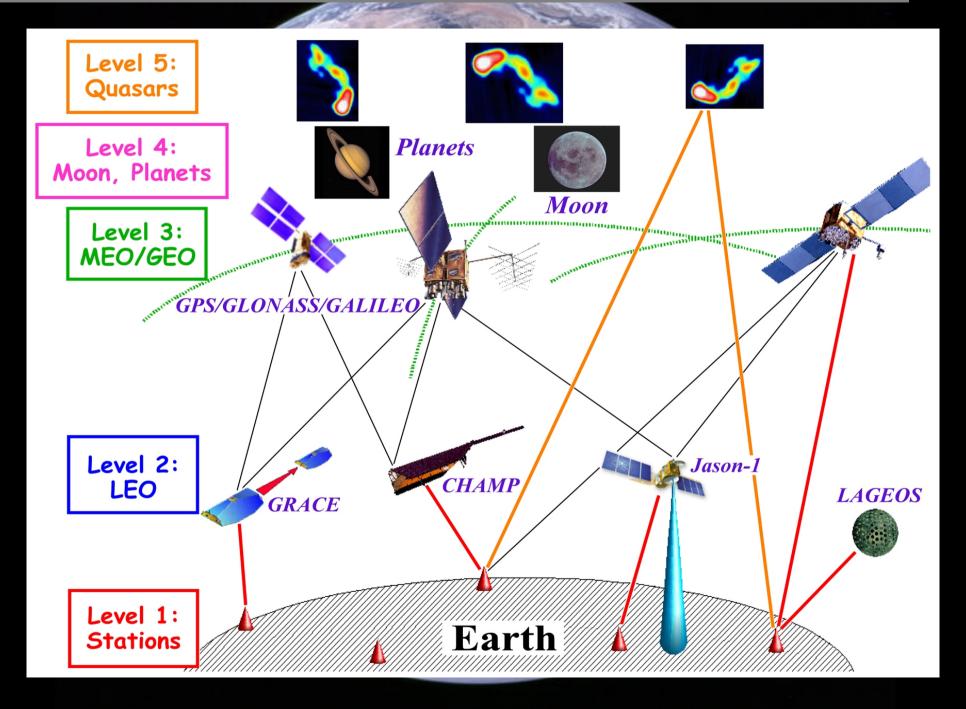
#### Monitoring the reference frame:

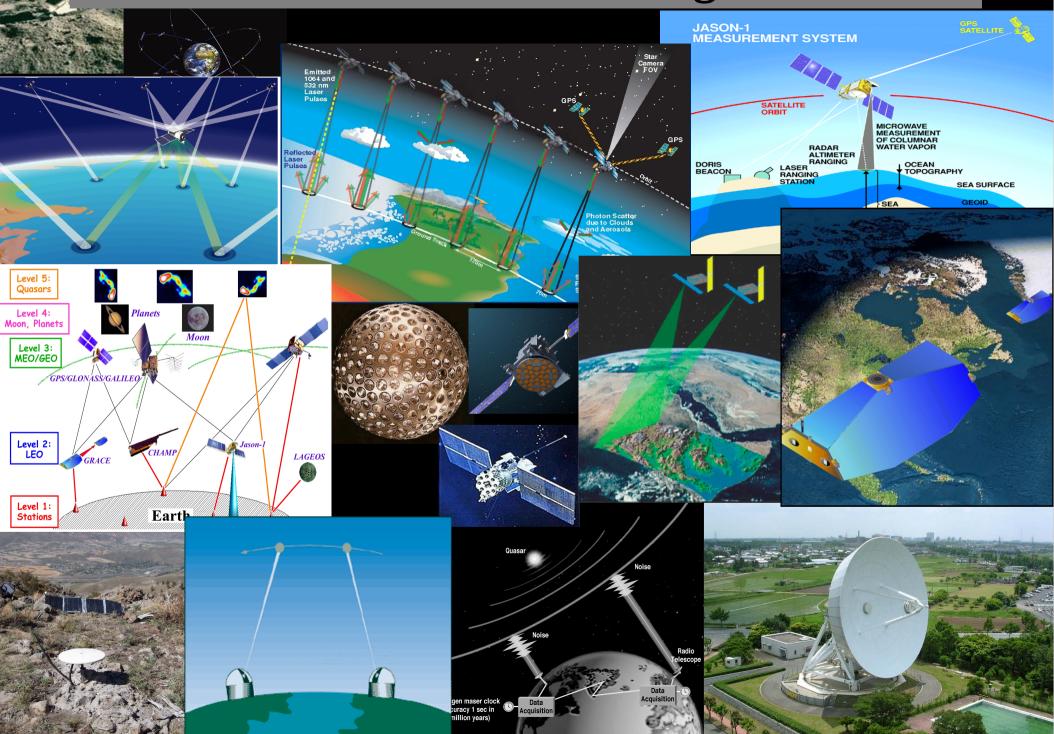


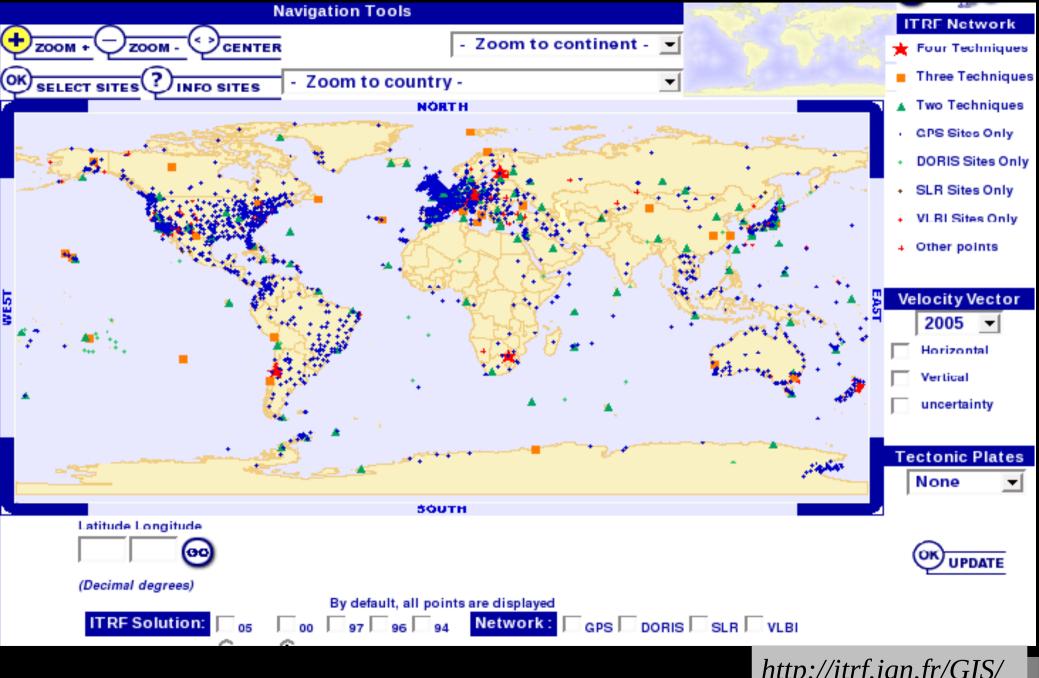
#### Monitoring the reference frame:

Component	Objective	Techniques	Responsibility
I. Geokinematics (size, shape, kinematics, deformation)	Shape and temporal variations of land/ice/ocean surface (plates, intra- plates, volcanoes, earthquakes, glaciers, ocean variability, sea level)	Altimetry, InSAR, GNSS-cluster, VLBI, SLR, DORIS, imaging techniques, leveling, tide gauges	International and national projects, space missions, IGS, IAS, future InSAR service
II. Earth Rotation (nutation, precession, polar motion, variations in length-of-day)	Integrated effect of changes in angular momentum and inertia tensor (mass changes in atmosphere, cryosphere, oceans, solid Earth, core/mantle; momentum exchange between Earth system components)	Classical astronomy, VLBI, LLR, SLR, GNSS, DORIS, under development: terrestrial gyroscopes	International geodetic and astrono- mical community (IERS, IGS, IVS, ILRS, IDS)
III. Gravity field	Geoid, Earth's static gravitational potential, temporal variations induced by solid Earth processes and mass transport in the global water cycle.	Terrestrial gravimetry (absolute and relative), airborne gravimetry, satel- lite orbits, dedicated satellite missions (CHAMP, GRACE, GOCE)	International geophysical and geo- detic community (GGP, IGFS, BGI)
IV. Terrestrial Frame	Global cluster of fiducial point, determined at mm to cm level	VLBI, GNSS, SLR, LLR, DORIS, time kee- ping/transfer absolute gravimetry, gravity recording	International geodetic community (IERS with support of IVS, ILRS, IGS, and IDS)

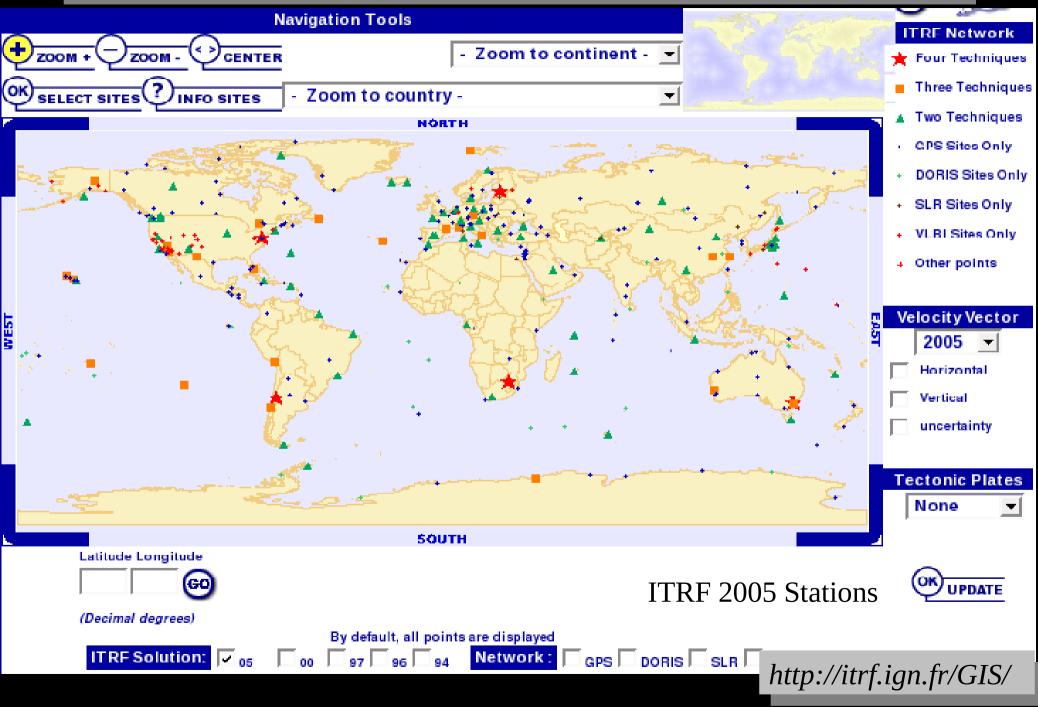








http://itrf.ign.fr/GIS/



# **Relation between ITRS and other Systems**

- For practical purposes, some global systems were introduced on national level. Of particular interest here is the World Geodetic System (WGS), which is still widely used in non-scientific applications.
- A first World Geodetic Reference System was in 1960 introduced by the U.S. Department of Defense (DoD) as WGS60 and later updated through WGS66 and WGS72.
- In 1984, the DoD introduced the World Geodetic System 1984 (WGS84). The first realization of the WGS84 was based on observations from the U.S. Navy Navigation Satellite System (Doppler Transit). This materialization of the WGS84 was achieved by aligning as closed as possible the DoD reference frame NSWC-9Z2 to the BIH Conventional Terrestrial System (BTS) at the epoch 1984.0.
- Estimated accuracy of 1-2 meters.

# **Relation between ITRS and other Systems**

Based on broadcast ephemeries, GPS receivers provide coordinates in the WGS84, and for many practical purposes such as air navigation, the WGS84 is in use.

Therefore, it is worthwhile to consider the current relation between WGS84 and the more accurate ITRF.

1994, the DoD made an attempt to align WGS84 with ITRF. For that, new coordinates for the ten DoD tracking stations were determined at the epoch 1994.0 used GPS tracking data collected at these sites together with a subset of the IGS tracking stations, with the ITRF91 coordinates of the later stations being held fixed in the process. This refined WGS84 realization is denoted as WGS84 (G730), with the 'G' indicating that the frame is GPS derived and '730' denoting the GPS week number when the new coordinates where implemented by DMA in their orbit processing.

Moreover, the original WGS84 GM value was replaced by the value given in the IERS 1992 standards.

The introduction of this new frame for GPS resulted in more precise ephemeris in the GPS broadcast messages.

# **Relation between ITRS and other Systems**

The importance of the ITRF as the most accurate global reference frame is increasingly acknowledged outside the scientific community.

For example, the European Commission and the Government of the United States of America recently agreed to align the reference systems of Galileo and GPS as close as possible to the ITRS, in order to ensure the interoperability of the two GNSS.

# **Dynamical Reference Frames**

A Conventional Dynamical Reference Frame (CDTF) specifies the planetary and lunar ephemerides, which are required, among other purposes, to determine the tidal potential at any point in the solar system, to determine the barycentre of the solar system, and to compute the geometry of the space-time continuum in the solar system. Besides orbital parameters, the CDTF also specifies the mass values of the bodies in the solar system.

- The frame for planetary and lunar ephemeries are frequently updated, mainly by the JPL.
- Examples: Development Ephemeris DE405 and the Lunar Ephemeris LE405, respectively
- These ephemerides have been adjusted to all relevant observational data and are given in the ICRF.
- It is interesting to note that some of the planetary mass values have changed considerable from the older IAU 1976 values over the DE200 to the values used for DE405.