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GEOL 495
Homework 2

Problem 1: **What are key physical constants related to gravitation and the gravity field of a planet and how well are these known?**

The universal gravitational constant (*G*), the mass of the objects involved in the gravity field (m_n), and the distance of observation from the center of that object's mass (*r*), are the physical constants necessary to describe the gravity field of any mass. This force is inversely proportional to r^2, making it diminish rapidly with distance from the center of mass (CM). The mass of the earth is well known, but the position of Earth's CM and the value of *G* are not, and since the Earth is not an ideal ellipsoid, *r* also is not simple to determine.

Problem 2: **Name and characterize the main equations related to the gravity potential.**

Newton's Law of Gravitation is fundamental to describe a point masses gravitational potential, where the force of potential is represented: $\mathrm{F}=G\frac{m_1 m_2}{r^2}$, looking at the acceleration of gravity experienced by one of these masses (typically the lesser mass) Newton's Law becomes: $\mathrm{a}=g=\frac{Gm}{r^2}$. Since planetary objects are far more complex than a single point mass, the integral form of Newton's law more accurately calculates potential by modeling gravitational potential as a function of position outside the object (*x*) and accounting for many point masses and densities, added together to yield: $\Phi(x)=\int_{mass}\frac{\mathrm{G}\rho(x')}{\|x-x'\|}d^3x'$. Gravitational potential is also described as the flux in a potential field by Gauss' Law, derived from Newton's Law of Gravitation, giving the more accurate: $\oint_{\partial V}\vec{g}\bullet d\vec{\mathrm{A}}=\int_V(-4\pi G\rho)d\mathrm{V}$, where flux is integrated over, and orthogonal to the surface enclosing a mass. Further refinement describes natural situations better still, using normal gradient vectors to describe the potential field of rotating elliptical objects in both the Laplace and Poisson equations, respectively: $\nabla^2\Phi(\vec{x})=2\omega^2$ and $\nabla^2\Phi(\vec{x})=-4\pi G\rho(\vec{x})+2\omega^2$, which can be solved in spherical coordinates, and expanded to account for density and harmonic free oscillations.

Problem 3: **How large are the deviations of the geoid from the reference ellipsoid and how are these deviations explained?**

The elevation of the geoid and the reference ellipsoid differ by about -110m and +85m because of a heterogeneous mass distribution throughout the planet. This mass distribution sets up a resonance on a rotating, wobbling, viscoelastic, ellipsoid; which is given by the equation: $N = \left.\frac{\Delta\Phi}{g_T}\right|_{r=a}$ which can be expanded to account for density and harmonic oscillations in spherical coordinates.

Problem 4: **Explain in simple words the origin of tides.**

Tides are generated by the periodic gravitational pull exerted on the Earth by the Moon, Sun, and to a lesser extent the rest of the solar system and extra-solar objects, along with the inertia from Earth's rotation.

Problem 5: **Why do we see ocean tides?**

The oceans are much less viscous, and more responsive to stress than the rigid earth, making the tidal bulge more apparent over water than land. Additionally, the geoid, atmospheric forces, bathymetry, and thermodynamic forces have an integrated effect on the readily deformable oceans, causing regional differences in the apparent tides.

Problem 6: **Why are the amplitude and phases of semidiurnal and diurnal tides varying irregularly in space?**

Tides are the integrated product of multiple gravitational forces acting on a system, on Earth the most significant forces are caused by the Sun and Moon. Since the period of solar day and year are not synchronous with the Moon's, and the tidal bulge on a sphere occurs both towards and sway from the acting body, tides vary according to the position of all considered bodies. Irregular tidal patterns occur due to the variant distance between all considered bodies, the inclination of the Moon's orbital plane from the ecliptic, and Earth's own changes in orientation.

Problem 7: **How large is the largest equilibrium tide on Earth?**

In a static Earth-Moon system, the tidal deformation of the respective bodies would equalize, and the tidal bulge would reach a theoretical maximum. Being a static system implies that parameters such as harmonic free oscillations, structural heterogeneities, deformability period, angular velocity, and relative angles of the considered bodies are not considered; making for a simpler, but less accurate model. Using the constants: g=9.82ms^{-2}, G=6.673×10^{-11}m^{3}kg^{-1}s^{-2}, M_{moon}=7.3477×10^{22}kg,

$s=3.84403\times10^8$km, $rs^{-1}=1.6\times10^{-2}$, and $l_{max}=6$, I solve for the equilibrium tide using the formula: $\delta N = \frac{V}{g}$, where tidal potential is: $V = \frac{GM_{moon}}{s} \cdot \sum_{l=2}^{\infty} \left(\frac{r}{s}\right)^l P_l\left(\cos\psi\right)$, since a static system is assumed I will consider the geocentric zenith angle to be 0°, causing the Legendre Polynomial term to become 1 and drop out giving: $\delta N=337.9$m.

Problem 8: **Why does the Moon keep the same face toward the Earth?**

The Moon's eccentric center of gravity is attracted to the barycenter of the Earth – Moon system, exaggerated by the inclination between the equatorial plane and the plane of the Moon's orbit. Over time the system's motion slowed to become tidally locked, until the Moon's period of axial rotation and its period of orbital revolution about the Earth were synchronous.

Problem 9: **How large is the tidal bulge of the Moon?**

The principal lunar semi-diurnal tide is the M2 wave, with the most significant displacement of all tidal constituents at 385mm vertically, 58mm horizontally, occurring at 12.42hour intervals.

Problem 10**: What are the main rotational eigenmodes of the Earth and to which parts of the Earth are they mainly attributed?**

Harmonic oscillations at multiple frequencies are produced by heterogeneities in mass and in viscosity of layers within the rotating Earth, combined with angular torque generated by tidal forces, create three main eigenmodes. The Chandler wobble is attributed to and damped by the mantle's viscous response, maintained by seasonal atmospheric and hydrospheric mass redistribution, along with tectonic shifts and earthquakes. The annual wobble is a nutation of the fluid core, with a period determined by the ellipticity of the core mantle boundary. The free inner core nutation is the result of interactions between the dissimilar layers of the inner and outer core.