GEOL 700(i): Geophysical Geodesy - Day 5

Preliminaries

- Issues with Problem Set #2? Questions? Revelations?

- Flesh et al., 2000, led by Jay.

New Readings

- STRAIN: Turcotte and Schubert, Segall

Note you can now read Paul Segall's book “Earthquake and Volcano Deformation” online via the UNR library web page.

go to http://www.knowledgecenter.unr.edu/

under Start Searching type in name of the book, hit return. Click on the right choice, note it says “: Reconstruction German Histories” .... not sure why....
click on “Full-text version (UNR users only)”.

Strain, and Strain Rate

Eulerian vs. Lagrangian descriptions of deformation.

In two-dimensions:

The basics in changes of shape.

$$\varepsilon_{xx}=\frac{\partial u_x}{\partial x} \quad , \quad \varepsilon_{yy}=\frac{\partial u_y}{\partial y} \quad , \quad \varepsilon_{xy}=\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x} \quad .$$

Tensor shear strain: $$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

Engineering shear strain: $$\varepsilon_{ij}=\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

Two different conventions. Remember to know which one you are using.

Note $\varepsilon_{yx}=\varepsilon_{xy}$.

The strain (rate) tensor $\boldsymbol{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{yx}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{bmatrix}$ keeps track of all three.

Here, u_x is displacement so $\boldsymbol{\varepsilon}$ is strain. Alternatively u_x could be velocity, in which case $\boldsymbol{\varepsilon}$ would be strain *rate*.

Velocity gradients are not (necessarily) strain rates. For example, on a rotating plate, velocity gradients are not zero, but strain rate is zero. We must separate the displacement tensor into components of strain and rotation.

Given a velocity field that is smoothly varying, in a local patch, we can estimate a constant strain rate tensor value. This can be separated into rotation and strain parts if the network is "strong".

Examples of strong vs. weak networks.

Principal strain axes

These are the eigenvectors of the strain tensor i.e. the vectors that satisfy:

$$\boldsymbol{\varepsilon}\, \boldsymbol{dx} = A\, \boldsymbol{dx}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor, and A is a scalar. These two eigenvectors have directions which correspond to the principal strain rate axes. The magnitude of these two vectors are the principal strain rates ε_1 and ε_2 . ε_1 is usually the larger value so $\varepsilon_1 - \varepsilon_2$ is positive.

Strain invariants

There are properties of the strain tensor that do not change with changes in coordinate axes (e.g. rotations, and translations).

First Invariant:

$$\varepsilon_{xx} + \varepsilon_{yy} = tr(\varepsilon) = \varepsilon_1 + \varepsilon_2$$

The first invariant is always the trace of the tensor (in 2D or 3D). It corresponds to the dilatational (volumetric) component of the shape change.

Second Invariant:

formally

$$\varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}^2$$

Another form is commonly used (C. Kreemer personal communication) which has the advantage of being in units of strain (not strain squared):

$\sqrt{\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + 2\varepsilon_{xy}^2}$ (sometimes shown squared)

This contains shear and dilatation components of deformation.

Also the difference note that in terms of principal strains:

$$\varepsilon_1 - \varepsilon_2$$

is invariant to coordinate axis changes. It represents shear strain because if the principal strains are equal (isotropic dilatation) then the shear is zero. If $\varepsilon_1 = -\varepsilon_2$ then dilatation is zero and shear is $2\varepsilon_1$.