Geophysical Geodesy GEOL 700(i)

Day 3: September 6, 2011

Beginnings

• Request to start class at 10:45

• Discussion of Problem Set #1
tribulations, hangups, successes...everybody up and running?

• Discussion of Coblentz et al., 1994, led by Paul

X,Y,Z to latitude, longitude and height

For details see e.g. WGS4 manual, v2.4 February 12, 1998. Details on Reading List.

Also, if you are curious see closed form solutions that Jay provided.

See MATLAB scripts on class page (not there yet) to perform various transformations.

XYZ to Latitude, Longitude and Height

$$\phi = \arctan\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\left(1-e^2\frac{\nu}{\nu+h}\right)^{-1} \quad \text{latitude}$$

$$\lambda = \arctan\left(\frac{Y}{X}\right) \quad \text{longitude}$$

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\phi} - \nu \quad \text{height}$$

where $\nu = \frac{a}{\sqrt{(1-e^2\sin^2\phi)}}$ is "radius of curvature in the prime vertical"

a is semi-major axis and *e* is eccentricity $e^2 = f(2-f)$ and are known beforehand.

f is usually ~1/298 on Earth, exact value depends on ellipsoid used (e.g. WGS4)

Problem: must know *h* and ϕ to start, so solve *iteratively*.

X,Y,Z to N,E,U, velocities and displacements

Draw the situation.

Right hand rule. North East Down vs. North East Up.

MATLAB scripts

The following example uses North, East, Down coordinates, but could be easily modified to East, North, Up. Note coordinate axes always obey right hand rule.

$$\boldsymbol{V}_{ned} = \boldsymbol{T}\,\boldsymbol{V}_{xyz}$$

where $T = \begin{bmatrix} Tnx & Tny & Tnz \\ Tex & Tey & Tez \\ Tdx & Tdy & Tdz \end{bmatrix}$

and

$$\boldsymbol{V}_{xyz} = \boldsymbol{T}^{-1}\boldsymbol{V}_{ned}$$

and

$$Tnx = \boldsymbol{n}\cdot\boldsymbol{x} = -\sin\phi\cos\lambda$$

$$Tex = \boldsymbol{e}\cdot\boldsymbol{x} = -\sin\lambda$$

$$Tdx = \boldsymbol{d}\cdot\boldsymbol{x} = -\cos\phi\cos\lambda$$

$$Tny = \boldsymbol{n}\cdot\boldsymbol{y} = -\sin\phi\sin\lambda$$

$$Tey = \boldsymbol{e}\cdot\boldsymbol{y} = \cos\lambda$$

$$Tdy = \boldsymbol{d}\cdot\boldsymbol{y} = -\cos\phi\sin\lambda$$

$$Tnz = \boldsymbol{n}\cdot\boldsymbol{z} = \cos\phi$$

$$Tez = \boldsymbol{e}\cdot\boldsymbol{z} = 0$$

$$Tdz = \boldsymbol{d}\cdot\boldsymbol{z} = -\sin\phi$$

see Cox, A., and R. B. Hart (1986), *Plate Tectonics: How it Works*, 392 pp., Blackwell Scientific Publishing, Brookeline Village, MA. Box 4-2, p. 155-156.

And very conveniently, to get the uncertainties

$$\boldsymbol{C}_{ned} = \boldsymbol{T}\,\boldsymbol{C}_{xyz}\,\boldsymbol{T}'$$

where the diagonals of C_{ned} are the uncertainties of v_n v_e , v_d . Very useful!

XYZ displacements to NEU displacements

same just no /dt

Helmert 7-parameter transformations

Translation - *T*

Rotation - *R*

Scale - *s*

x' = sR**x** + T

<u>Rotation</u>

The square 3x3 matrix ***R*** rotates a 3x1 vector into another 3x1 vector. If ***R*** is a rotation matrix it does not change the length of ***x***, only its orientation. It is, in general, an orthogonal matrix ($\boldsymbol{R}^{-1} = \boldsymbol{R}^T$) with det ***R*** = 1.

$$\boldsymbol{x}' = \boldsymbol{R}\,\boldsymbol{x}$$

one example of a rotation matrix is:

$$\boldsymbol{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix} ,$$

which rotates a vector around the x-axis. In general you need three numbers to specify a rotation. There are equivalents for $\boldsymbol{R}_y$ and $\boldsymbol{R}_z$ which can be multiplied together to get any particular rotation.

<u>Translation</u>

This one is pretty simple, just add a vector d***x*** to ***x***

$$\boldsymbol{x}' = \boldsymbol{x} + d\,\boldsymbol{x}$$

<u>Scale</u>

scale changes are usually small, measured in parts per million or billion in global geodetic problems.

$$\boldsymbol{x}' = (1+s)\,\boldsymbol{x}$$

where s << 1
Put it all together to get:

$$\boldsymbol{x}' = d\,\boldsymbol{x} + (1+s)\,\boldsymbol{R}\,\boldsymbol{x}$$

Note that here R has the form

$$\boldsymbol{R} = \begin{pmatrix} 1 & -r_z & r_y \\ r_z & 1 & -r_x \\ -r_y & r_x & 1 \end{pmatrix} \;,$$

These transformations are used to change from one reference frame to another.

also see e.g.:

http://mathworld.wolfram.com/RotationMatrix.html

http://en.wikipedia.org/wiki/Helmert_transformation

for more information

14 Parameter Helmert Transformation

same but with *dp/dt* of all parameters *p*

Euler Rotations on a Sphere

How do we describe the motion of a tectonic plate on Earth? A rotation.

Theory of plate tectonics

From a position on surface, and Euler rotation vector

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{x}$$

picture it.

Cross product versus matrix forms of rotation

$$\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix} = \begin{pmatrix} -\omega_z y + \omega_y z \\ \omega_z x - \omega_x z \\ -\omega_y x + \omega_x y \end{pmatrix} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{W}\boldsymbol{x}$$

Note that ***W*** looks a little bit like ***R***, but differs along the diagonal.

W is ofter represented as a latitude, longitude and magnitude (rate) of rotation.
e.g. *Altamimi et al.*, (2007), *Kreemer et al*, 2000.

Reference Frames

Where are you? How fast are you going?
Where is the center of the Earth (to within 1 mm and 1 mm/yr)?

Generally we are concerned with a class of reference frame known as Earth-Centered, Earth-Fixed (ECEF)

International Terrestrial Reference Frame (ITRF), what is it? A model.

North America Fixed Frame: e.g. SNARF

CE, CF, CM

<u>Types of Global Frames</u>

CM - Center of Mass of entire Earth System, including atmosphere and oceans. Its everything. Center of CM does not change its trajectory through inertial space. Need Satellite Laser Ranging (SLR) to access the CM. But there are not that many SLR satellites. ITRF 2005 is a CM frame. Accuracy to which the origin of the CM frame is known is on the order of 1-2 mm/yr. Latest version ITRF2008 has been recently released.

CE - Center of Mass of the Solid Earth (excludes atmospheres and oceans). Loads from fluids on the Earth surface load and deform the solid Earth so this changes the location of the CE with respect to the CM.

CF - Center of Figure. Geometric center of the outer envelope of the Earth shape. No net translation
frame. Does not account for mass distribution in any way since its just based on the outline.

CL - Center of surface lateral figure. Similar to CF but based mostly on horizontal motions, rather than all three (x,y,z or n,e,u).

CH - Center of surface height figure. The opposite of CL with respect to its relationship to CF. The frame constrained to have no net vertical motion.

No net rotation of the solid Earth (NNR) reference frame.

The International Terrestrial Reference Frame 2005 (ITRF 2005: Altamimi et al., 2007) is a NNR CM frame. Based on GPS, SLR, VLBI, DORIS *combinations*.

ITRF is a product based on a lot of data and a few assumptions. For some it is a product (a model), and for other it is a foundation. It is essentially a text file with a list of coordinates and rates for GPS and other types of sites around the world. So however we arrive at our own solutions for coordinates we can transform our solution into ITRF, as long as we know the rate and motion of our sites with respect to stations in the ITRF.

Ambiguity of vertical displacement.

Systems like GPS lack sensitivity to scale of geometry

Types of Velocity Reference frames we sometimes use

Western U.S. GPS velocity field

wrt ITRF2005
wrt North America
wrt Sierra Nevada

Baselines

SNARF - GIA corrected. Similar to ITRF except different...
Other Reference Frame ideas?

Fiducial vs. non-fiducial Frame
Where all the measurements are expressed with respect to certain site (or sites) in a network.