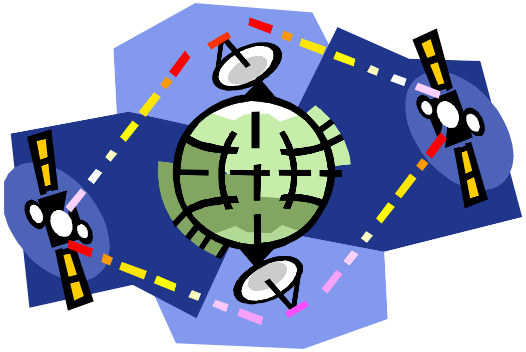
GPS the Interdisciplinary Chameleon:
How Does it do That?



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Cool Science using GPS

Application	Uses GPS to Estimate...
Tectonophysics	Earth's surface as slowly deforming polyhedron
Seismology	< 1 Hz time series of station position Regional strain variation during earthquake cycle
Atmospheric Science	Angular momentum balanced by Earth rotation In-situ water vapor (vertical integral & gradient) Stratospheric temperature (by low Earth orbiters)
Space Weather	Ionospheric total electron content (in-situ and remote) Ionospheric scintillations (electrical storms)
Hydrology	Subsidence due to ground water variation Surface mass redistribution using load dynamics Surface mass redistribution using Earth rotation
Oceanography	Sea surface height (SSH) by GPS altimetry Sea surface state by GPS reflections (down looking) In-Situ Correction of Tide Gauge Record Static sea level using load dynamics
Gravimetry	Geopotential variations using load dynamics

Outline

- Some basic terms
- What is being measured?
- What is being modeled?
- GPS positioning methods and typical precision
- What about military degradation?
- What drives the data rate?
- A few misconceptions

Some Basic Terms

- **Geodesy**
  - "The science of Earth's time varying shape and gravity field"
  - Typically characterized by
    - measurement error (<1 mm) < observation model error
    - today's noise = tomorrow's signal ("Geodesy is like an onion…")
- **GPS - Global Positioning System**
  - Constellation of ~30 satellites with atomic clocks
  - Transmit timing signals on L-band carriers (1.57542, 1.2276 GHz)
  - User positioned by 4+ satellites visible anywhere, anytime
- **GNSS - Global Navigation Satellite System**
  - GPS, GLONASS, GALILEO, COMPASS,…
- **IGS - International GNSS Service**
  - Provides GPS data products to serve geophysical applications
- **IERS - International Earth Rotation & Reference Frames Service**
  - Provides reference system conventions and data products
  - ITRF - International Terrestrial Reference Frame (site coordinates)

The GPS Signal

- GPS signal tells a receiver the satellite clock time
  - GPS receiver compares clock times: receiver – satellite
  - Hence time of flight, hence range, hence receiver position
- GPS signal driven by atomic clock on each satellite

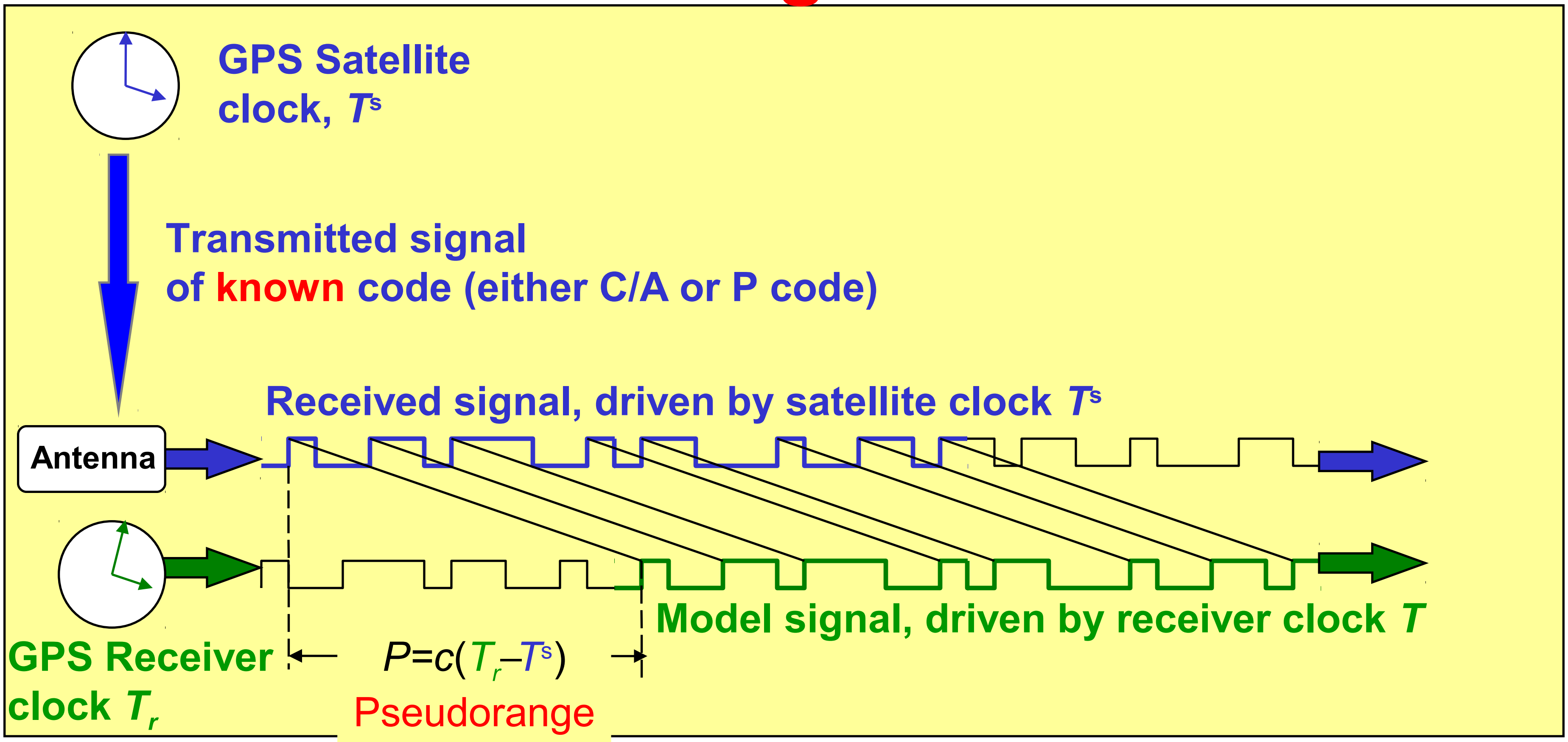
  Clock frequency = 10.23 MHz

  (Set intentionally lower than this to account for relativistic effects)
- Two carrier signals (sinusoidal) are coherent

  L1 = 154 x 10.23 MHz   wavelength = 19.0 cm

  L2 = 120 x 10.23 MHz   wavelength = 24.4 cm
- Bits (+1 and -1) encoded on the carrier tell the time
  - Course C/A code on L1 - satellite time (C1)
  - Precise P code on L1 and L2 - satellite time (P1 and P2)
  - Navigation Message - satellite position, satellite clock bias, etc.

What is being measured?

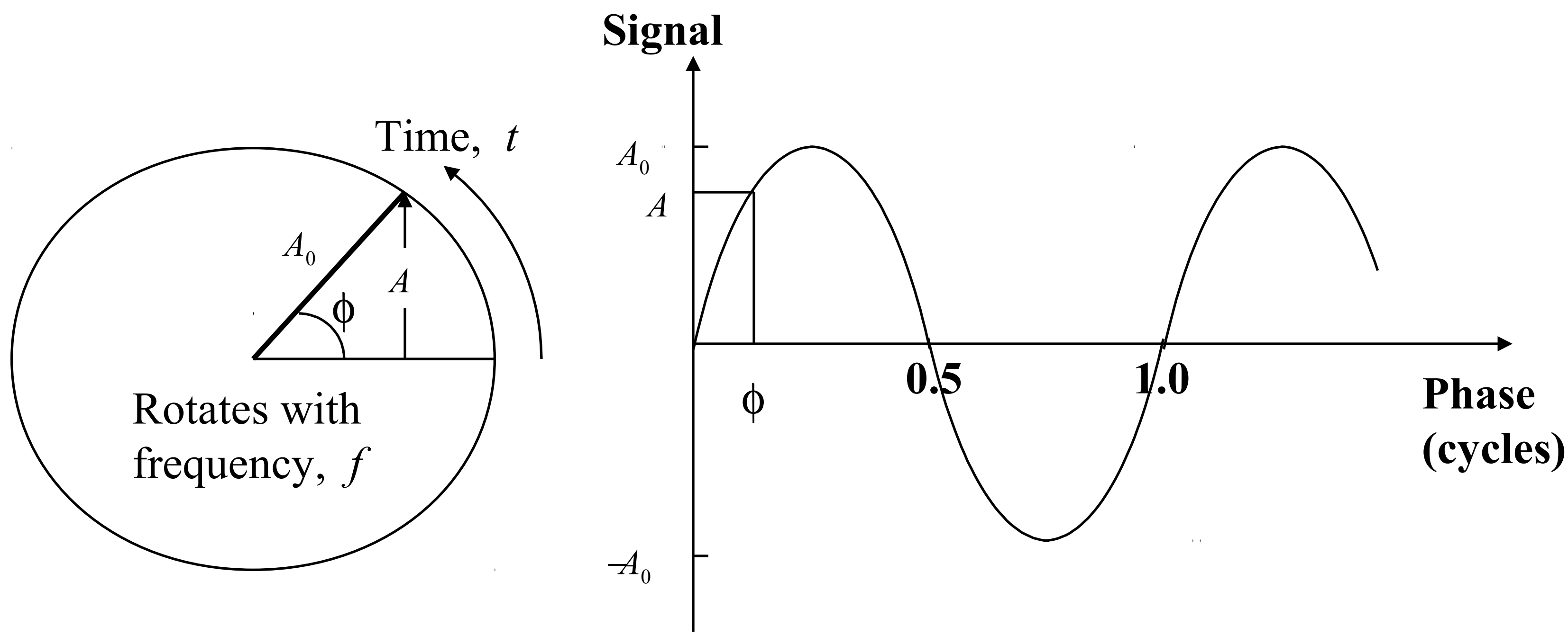


- GPS is actually a timing system
- Receiver firmware correlates received signal with replica model
- Observed time delay ×c is "pseudorange" (a biased range)

What is being modeled?

- Satellite clock
  - Clock error
  - Relativity (general + special)
- Satellite position
  - Gravity (Keplerian + higher degree)
  - Non-gravitational forces acting on the satellite (radiation pressure)
- Media delay
  - Vacuum delay (geometry, include general relativity)
  - Ionospheric dispersion (use dual-frequency data)
  - Troposphere refraction (signal speed and bending by Snell's law)
- Station position
  - Earth rotation (P.N.U.X.Y)
  - Solid Earth deformation (tides, loading)
  - Antenna diffraction (azimuth-elevation calibration)
  - Relative antenna-transmitter rotation (circular polarization effect)
- Station clock
  - Clock error

Meaning of Phase, ϕ



- **Steady rotation**

  $\phi = f\,t$ — phase ϕ (cycles), frequency f (Hz)

- **Sine wave:**

  $A = A_0 \sin(2\pi\,\phi)$ — amplitude A_0 = $A_0 \sin(\omega t)$

  angular frequency $\omega = 2\pi f$

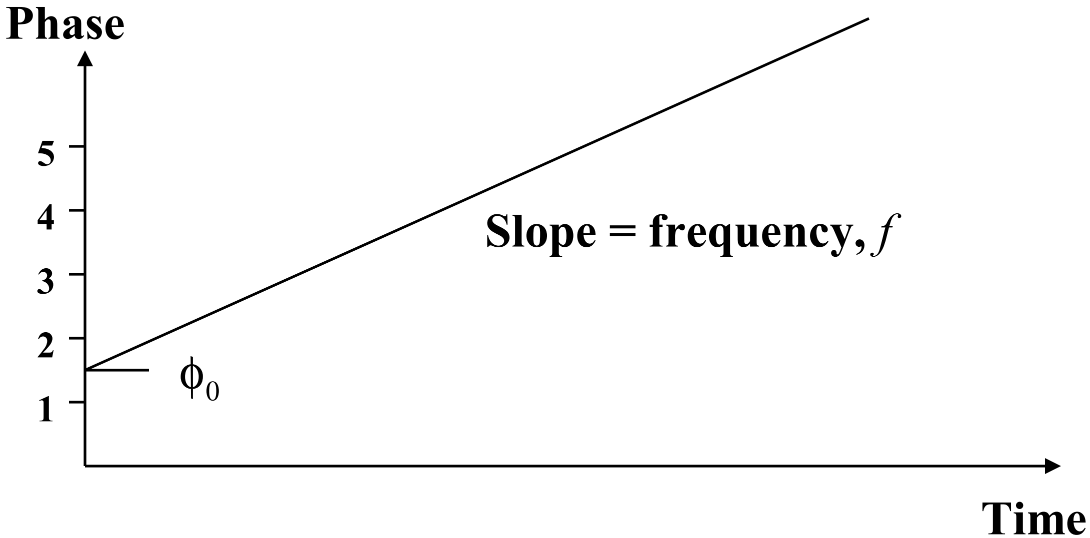
Meaning of Phase

- Therefore, for a sine wave
  - phase changes linearly in time:

$$\phi = f\,t \quad \text{phase } \phi \text{ (cycles), frequency } f \text{ (Hz)}$$

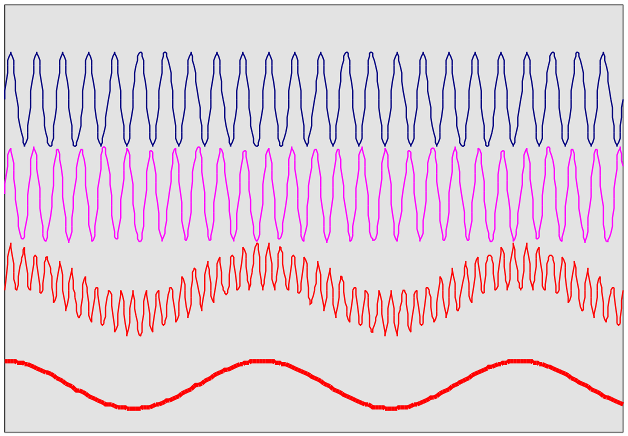
- Generally, the initial phase is not zero:

$$\phi = f\,t + \phi_0 \qquad \text{constant: } \phi_0 \text{ (cycles)}$$



What is Carrier Phase?

Receiver multiplies GPS satellite signal × reference signal from local oscillator



$$S(t) = S_0 \cos\varphi_S(t)$$

$$R(t) = R_0 \cos\varphi_R(t)$$

$$S(t)\times R(t) = \tfrac{1}{2} R_0 S_0 \left[\cos\left(\varphi_R - \varphi_S\right) + \cos\left(\varphi_R + \varphi_S\right)\right]$$

$$S(t)\otimes R(t) = \tfrac{1}{2} R_0 S_0 \cos\left(\varphi_R - \varphi_S\right)$$

"Baseband filter" removes high frequency

"Carrier phase" = phase of baseband signal (Ambiguity)

Carrier phase measurement:	$\Phi = \left(\phi_R - \phi_S\right) + N$
Model as satellite-receiver clock:	$\Phi = f\left(T_R - T_S\right) + N$
Convert cycles to range	$L = \lambda\Phi = c\left(T_R - T_S\right) + N\lambda$

Measurement error < 1 mm

What is Carrier Phase? (II)

- But GPS signal is carrier × code (+1 or -1)
- Standard GPS pseudorange *uses* the bits
  - meter-level precision
- Carrier phase measurement *strips off* the bits
  - mm-level precision but biased by integer cycle ambiguity N
- Cycle slips (change in N)
  - When receiver loses phase lock, changes unknown value of N
  - can be repaired by various data editing techniques
- Ambiguity resolution (estimation of N)
  - Integer estimation is an entire field of research in itself !
  - Accurate estimation is essential
  - Real-time kinematic (RTK): pair of receivers do this in real-time

Analytical view of observations

– **4 observation types (units of range):**

L_1: Carrier phase on L1 channel, f_1 = 154 x 10.23 MHz

L_2: Carrier phase on L2 channel, f_2 = 120 x 10.23 MHz

P_1: Pseudorange on L1 channel

P_2: Pseudorange on L2 channel

– **Simplified observation equations:**

$$L_1 = \rho + Z/\sin\theta - 1.546\ I + \lambda_1 N_1 \qquad \lambda_1 = c/f_1 \cong 19.0\text{ cm}$$

$$L_2 = \rho + Z/\sin\theta - 2.546\ I + \lambda_2 N_2 \qquad \lambda_2 = c/f_2 \cong 24.4\text{ cm}$$

$$P_1 = \rho + Z/\sin\theta + 1.546\ I$$

$$P_2 = \rho + Z/\sin\theta + 2.546\ I$$

ρ = **geometry plus clock bias**

Z = **zenith tropospheric delay**

$1/\sin\theta$ = **"mapping function" of elevation angle**

I = **dispersive ionospheric delay (P2-P1)**

N = **integer cycle ambiguity**

Why "Dual Frequency" Data?

- Dual frequency more expensive but often essential
  - "geodetic" quality receivers currently ~$10K

1. To estimate ionospheric delay (TEC)
   - (P1 – P2) proportional to Total Electron Content
   - Geometry and clock errors cancel
   - Instrumental "P1 – P2 bias" documented by IGS
1. To difference away ionospheric delay (0.1-10 meters)
   - assume delay proportional to $1/f^2$
   - "ionosphere-free" carrier phase LC = 2.546 L1 – 1.546 L2
   - residual ionospheric delay from higher order effects ~ mm
1. Automatic, robust data processing
   1. Compare all 4 data types (P1, P2, L2, L2) for consistency
   2. Cycle slip detection and correction
   3. Ambiguity resolution

GPS Positioning Methods and Typical Precision

- **Pseudorange positioning**
  - hand-held GPS, few-meter
  - hand held GPS receiving differential corrections, 1-meter
  - differential pseudorange with "carrier smoothing", 10-cm
  - limited by multipath errors
- **Dual-frequency carrier phase positioning**
  - hand-held GPS using RTK base station, 1-cm relative
  - geodetic GPS, 2-3 mm horizontal, 7-mm vertical (in global frame)
  - geodetic GPS (regional), 1-2 mm horizontal, 3-5 mm vertical

What about Military Degradation?

- No problem
- **Anti-Spoofing (A/S)** secret W code × P code
  - C/A code not encrypted, so can produce pseudorange (C1)
  - Encryption is identical on two frequencies! (P1 – P2)
  - Bit rate of the W code slower than P code (can use to get P2)
- **Selective Availability (S/A)** dithers satellite clock
  - Countered by usual geodetic processing methods
  - Or by real-time transmission of error from fixed “base” station
  - S/A turned off [*President Clinton*, 1 May 2000]
    - “encourage acceptance & integration of GPS into peaceful civil, commercial & scientific applications worldwide”
    - “encourage private sector investment in, and use of, U.S. GPS technologies & services.”

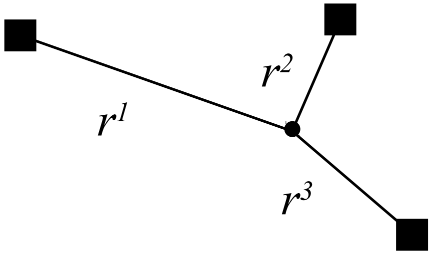
What Drives the Data Rate?

- Observation equations can typically be inverted at every measurement epoch
  - So long as sufficient satellites in view (typically 5)
  - So long as ambiguity resolution is possible
- Required solution rate can drive the data rate up
  - Navigation: typically 1 Hz
  - Seismic Strong Motion: ~ 10 Hz
    - For very high data rates, measurement noise becomes large
- Cost / Benefit can determine an optimal data rate
  - Geodetic: typically ~ 15 or 30 seconds
    - Typical position solution rate is every 24 hours
    - Typical tropospheric delay rate is every 5 – 60 minutes
    - Lower data rate creates data editing problems
    - After editing, data can be decimated to every 5 minutes
    - Data have correlated errors over few minutes

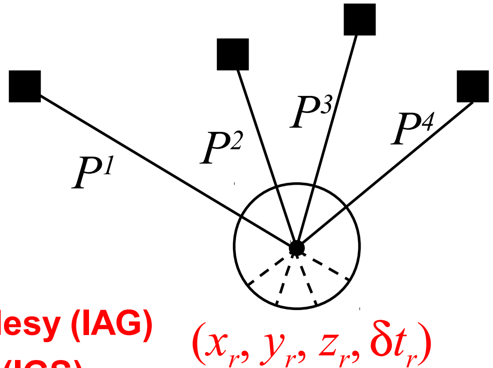
Key Idea I

- ***Applications relate to unknown parts of the model***
  - Positioning (hence motion by time series)
    - spatial registration - sensor position/attitude; altimetry; mapping, GIS
    - secular motion - tectonics; isostasy; plate boundary deformation
    - tidal motion - solid Earth tides; pole tide; ocean tidal loading
    - non-tidal loading - atmosphere; continental hydrology; ocean
    - earthquake cycle - co-seismic; post-seismic; inter-seismic
  - Timing
    - time registration (time tags) - GPS; satellite laser ranging; seismic
    - event triggering: 1 pulse per sec signal (PPS)
    - clock synchronization - defining Universal Coordinated Time (UTC)
  - Earth rotation (orientation) parameters (ERP or EOP)
    - Precession, nutation, universal time UT1 , polar motion
  - Precipitable water vapor (PWV)
  - Ionospheric total electron content (TEC)

Example: Point Positioning



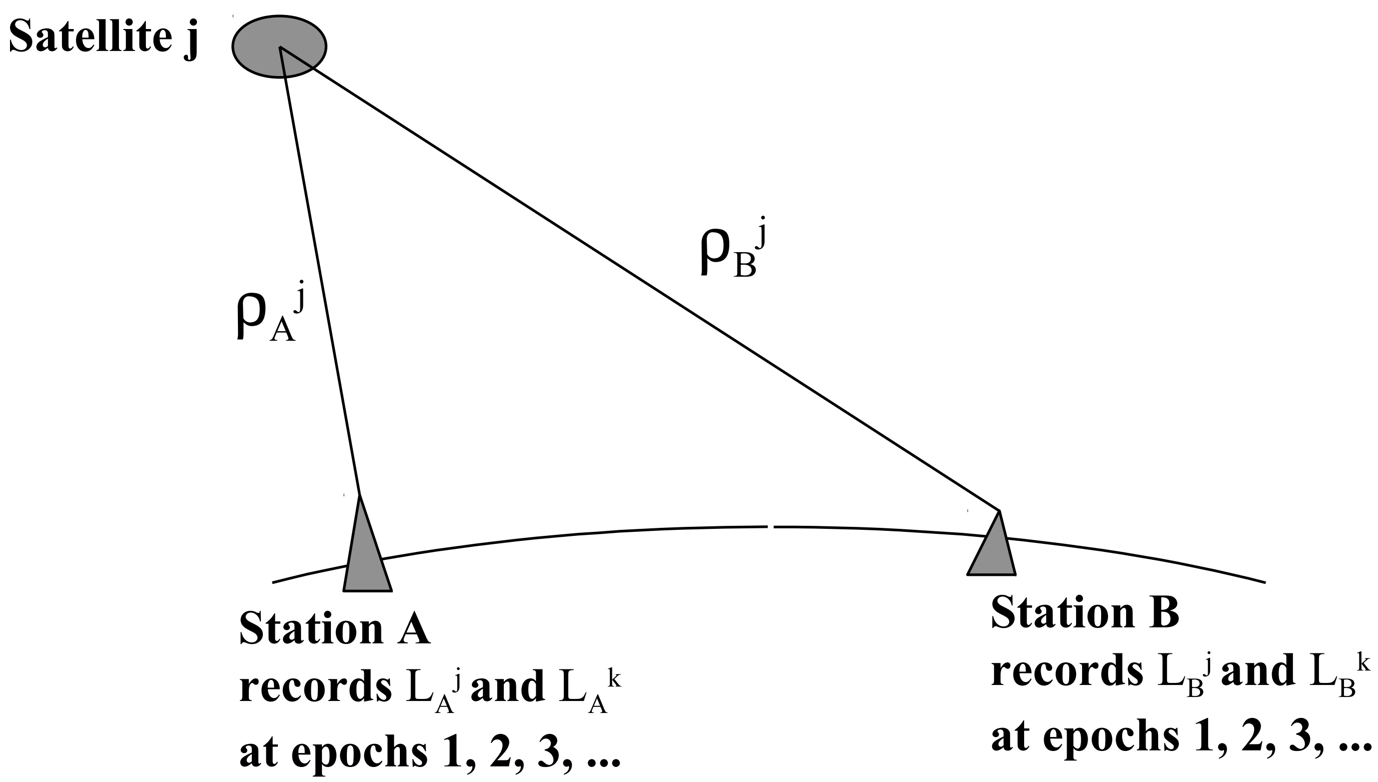
- **Similar to standard trilateration**
  - **3 ranges from 3 known points**
  - **Estimate 3 unknown point coordinates**
- **GPS point positioning**
  - **4 pseudoranges from 4 satellites**
  - **Estimate 4 parameters:**
    - **position (x_r, y_r, z_r) and**
    - **clock error (δt_r)**
  - **Other "known" parts of the model?**
    - **Real-time information from broadcast Navigation Message**
    - **Precise information from International Association of Geodesy (IAG)**
      - **GPS orbits and clocks from International GNSS Service (IGS)**
      - **Earth model from International Earth Rotation & Reference Frames Service (IERS)**
      - **Station positions from IERS Terrestrial Reference Frame (ITRF)**



Key Idea II

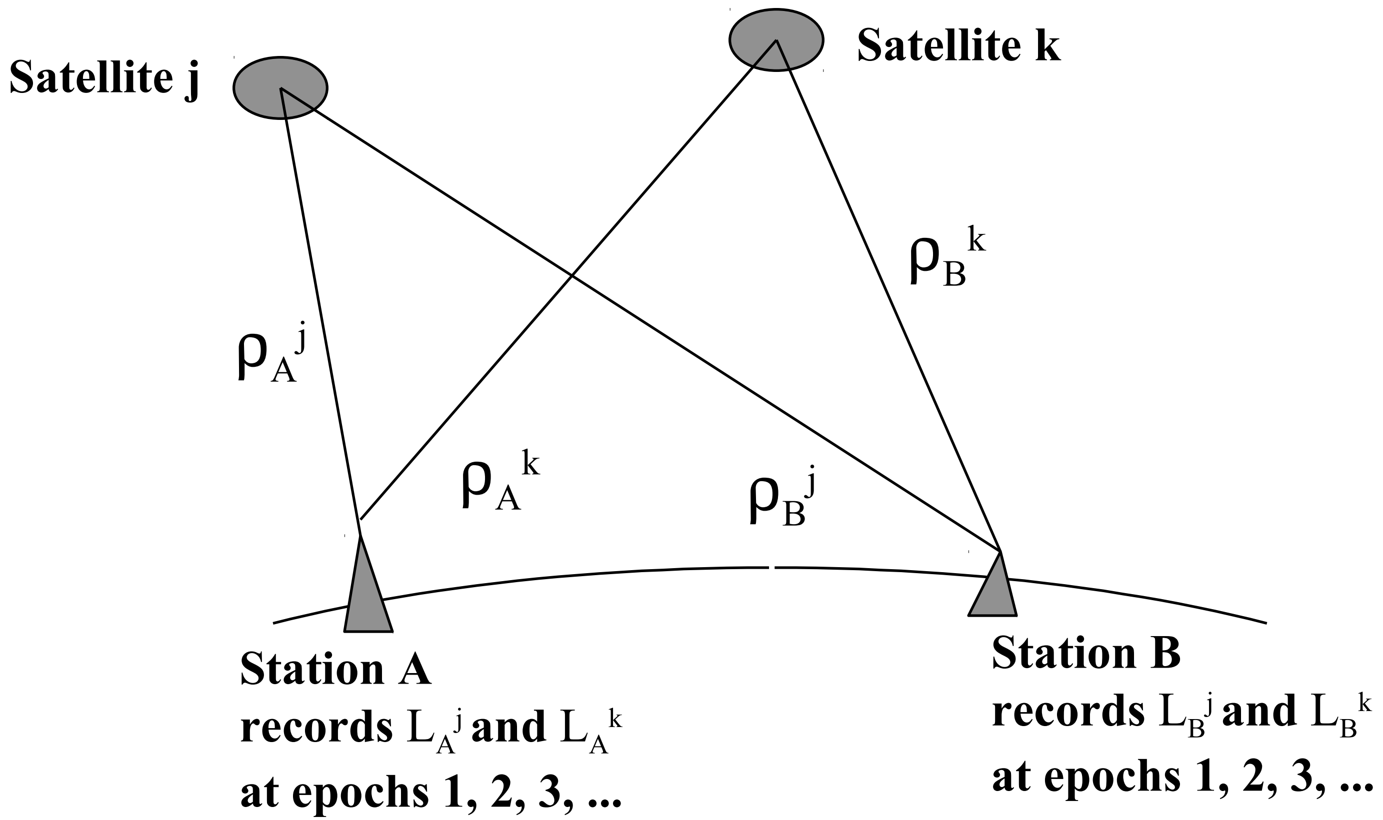
- ***Interpretation of the "unknowns" requires accurate treatment of "knowns"***
- **Methods** (generally a mixture)
  1. Assume models and other data (IERS, IGS, Nav. Message)
  2. Simultaneous estimation of "nuisance parameters"
     - We need precise position so we also estimate clock error
     - We need precise timing but we also estimate position
  3. Form linear combinations of data that eliminate the unknowns
     - Dual frequency combination eliminates most ionospheric delay
     - Double-differenced data eliminates station-satellite clock error
     - Baselines (relative point positions) reduce common-mode errors
  - Note: (2) and (3) give identical results
     - (2) is more time consuming, but provides spin-off results

Single Differencing



Eliminates satellite clock errors

Double Differencing

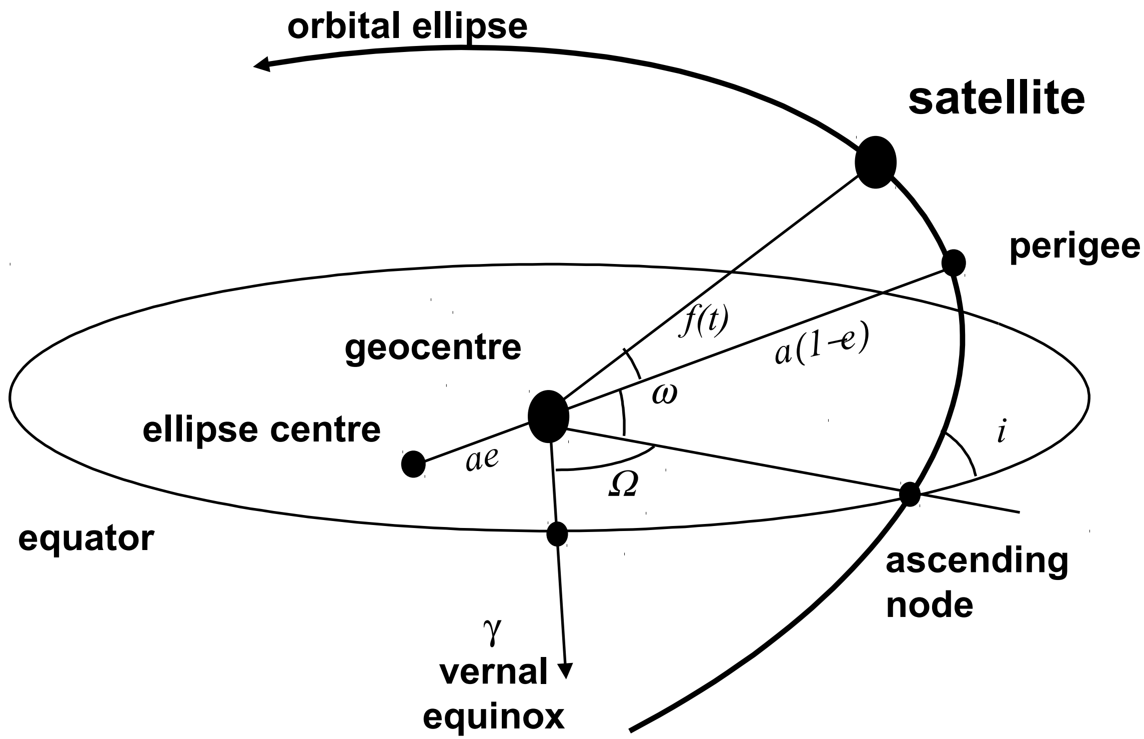


Eliminates satellite clock errors and station clock errors

Reduces orbit errors if stations are in same region

Relative positioning is more precise for stations that are closer

Example: "Known" Satellite Positions



- "Normal orbit" represented by 6 Keplerian elements:

a	semi-major axis	ω	argument of perigee
e	eccentricity	Ω	Right Ascension of ascending node
I	inclination (to equator)	$f(t)$	true anomaly at time t

But Real Orbits are Perturbed by Other Forces

	Perturbing Forces	Acceleration ($m\ s^{-2}$)	Error after 1 day (m)
Gravitational	Non-sphericity: oblateness	5×10^{-5}	10,000
	Non-sphericity: other	3×10^{-7}	200
	Moon	5×10^{-6}	3,000
	Sun	2×10^{-6}	800
	Solid Earth tides	1×10^{-9}	0.3
	Ocean tides	5×10^{-10}	0.04
Non-Grav.	Solar radiation pressure	6×10^{-8}	200
	Y-bias (misaligned panels)	5×10^{-10}	1.4
	Albedo (from Earth)	4×10^{-10}	0.03
	Drag, magnetic forces, etc...	$<< 10^{-10}$	0

From Herbert Landau, Ph.D. Thesis, 1988

Hence: Osculating Keplerian Elements

- **Extra Parameters added in Navigation message**

– Keplerian parameters at some reference time	$(a, e, i, \omega, \Omega, M_0)$
– correction to the mean motion	Δn
– rate of change of inclination	di/dt
– rate of change of node's right ascension	$d\Omega/dt$
– cosine and sine terms for inclination	$C_{ic} C_{is}$
– cosine and sine terms for radius	$C_{rc} C_{rs}$
– cosine and sine terms for argument of perigee	$C_{uc} C_{us}$

- **GPS Ephemerides**
  - Navigation Message is updated every hour
  - Receiver accounts for these extra terms. After all of this….
  - Resulting satellite position only accurate to few meters !!!
  - INTERNATIONAL GPS SERVICE (IGS) orbits: few centimeters

A Few Misconceptions (I)

DESIGN: "Network configuration is crucial for precise positioning"

- NOT true except for the global-scale network
- ONLY important for spatial resolution of the geophysical signal

EQUIPMENT: "For highest accuracy, every station should have

… an atomic clock"

- Clock bias is either estimated or differenced away

… an accurate barometer"

- Barometers do NOT significantly improve estimation of tropo delay

… a water vapor radiometer (WVR)"

- Marginal improvement; serious WVR errors during precipitation

"GPS can do everything"

- Cumulative orbit instability in inertial space (precession, nutation, UT1)
  - Requires Very Long Baseline Interferometry (VLBI)
- Inadequate models of non-grav. forces (locating Earth center of mass)
  1. Requires Satellite Laser Ranging (SLR)

A Few Misconceptions (II)

ACCURACY: “Choose method that minimizes the daily scatter”

- REAL surface loading varies station height by ~10 mm

QUALITY: “Small formal errors indicate a high quality solution”

- Does NOT address data quality, systematic error

LOW ELEVATIONS: “Low elevation data should not be used”

- Low el. data helps separate station height from zenith tropo delay

DATA SPAN: “To filter seasonal error, use *N*-year data-span

- For site velocity, half-integer year is optimal (integer is worst!)
- For velocities, recommend at least 2.5 years of data

Conclusions

- **GPS is a timing system**
  - Pseudorange (robust)
  - Carrier phase (precise)
  - Dual frequency (for ionospheric delay)
- **Requires a comprehensive observation model**
  - Anything that affects the apparent satellite-receiver time
  - Satellite-receiver geometry, media delay, clocks
  - Unknown parts of model can either be:
    - Estimated
    - Differenced away by linear combinations of data
    - Modeled stochastically (at every data epoch)
- **Applications**
  - Relate to the UNKNOWN parts of the model
  - Which requires accurate treatment of the KNOWN parts
  - Various services exist for the KNOWN parts (IGS, IERS,…)