GEOL 695: Geophysical Geodesy - Day 7

Preliminaries

- Readings
- Questions about the problem sets
- New Problem set
- Mid-term one week from today

New Reading:

Turcotte, D. L., and G. Schubert (1982), *Geodynamics Applications of Continuum Physics to Geological Problems*, John Wiley & Sons, Inc., New York.
Section 3-1 to 3-8.

talk about it thursday.

Strain, Continued...

FYI: the two shear strain definitions:

Tensor shear strain: $$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

Engineering shear strain: $$\varepsilon_{ij}=\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$$

remember: know which one you are using.

Elasticity - Entering the world of Dynamics

In geophysics:

“Kinematics” is the study of motions, deformation, geometric changes, the basic ingredients are measurements of distance and time.

“Dynamics” is the study of the forces (or principals) that drive or resist these motions.

Hooke's Law applies to a linear spring:

Force = - displacement X spring constant

$$\boldsymbol{F}=-k\,\boldsymbol{x}$$

(note vector quantities).

What does it mean? Elastic deformation is instantaneous and temporary. There is no time in this equation. Take the stress away and the material returns to its original shape, instantly (according to this model).

Further generalization and application to the deformation of solids.
Three dimensions, need them all. Now we need the concept of stress σ_{ij} (force per unit area) and relate it to strains in a general way:

$$\sigma_{ij}=c_{ijpq}\varepsilon_{pq}$$

where $c_{ijpq}=c_{jipq}$ (since $\sigma_{ij}=\sigma_{ji}$)

and $c_{ijpq}=c_{ijqp}$ (since $\varepsilon_{ij}=\varepsilon_{ji}$)

and $c_{ijpq}=c_{pqij}$ (thermodynamics).

which reduces the number of individual elements to from 81 to 21.

Furthermore we generally assume in elastic solids that the material is isotropic (not anisotropic) so that the orientation of the medium doesn't matter. This will further reduce the number of elastic parameters needed to 2.

$$\sigma_{ij}=\lambda\delta_{ij}\varepsilon_{kk}+2\mu\varepsilon_{ij}$$

where $\delta_{ij}=1$ when i=j and is zero otherwise.

Key elastic parameters.

The *shear modulus* μ , satisfies $\sigma_{12}=\mu\varepsilon_{12}$. Describes shear elasticity. (Note Turcotte and Schubert use *G*).

Young's modulus E , Describes uniaxial elasticity = tensile stress/tensile strain.

Lame's parameter λ , need it.

the B*ulk modulus K* , satisfies $p=K\Delta$

Possion's ratio ν describes the ratio between percent extension to percent contraction in the perpendicular direction when the material is subject to tension in one direction, but is free to deform in each of the other directions.

In almost all solid materials, the Possion's ratio is positive. What does it mean if Poisson's ratio is negative?

In the end all you need is two and you can get any of the others through simple conversions.

What are the units of these moduli?

Force Balance