GEOL 695: Geophysical Geodesy - Day 6

Preliminaries

- Attendance and Make up policies
- Readings

New Readings

required:
Turcotte, D. L., and G. Schubert (1982), *Geodynamics Applications of Continuum Physics to Geological Problems*, John Wiley & Sons, Inc., New York.
Section 2-7.

appendix of:
Savage, J. C., W. Gan, and J. L. Svarc (2001), Strain accumulation and rotation in the eastern California shear zone, *Journal of Geophysical Research*, *106*, B10, 21,995-922,007.

Shen-Tu, B., W. E. Holt, and J. A. Haines (1998), Contemporary kinematics of the western United States determined from earthquake moment tensors, very long baseline interferometry, and GPS observations, *Journal of Geophysical Research*, *103*, B8, 18,087-18,117.

Strain, Continued...

On a rotating plate, velocity gradients are not zero, but strain rate is zero.

in general $\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$

Principal strain axes. These are the eigenvectors of the strain tensor i.e. the vectors that satisfy:

$$\boldsymbol{\varepsilon}\, \mathbf{dx}=A\,\mathbf{dx}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor, and *A* is a scalar. These two eigenvectors have directions which correspond to the principal strain rate axes. The magnitude of these two vectors are the principal strain rates ε_1 and ε_2 . ε_1 is usually the larger value so $\varepsilon_1-\varepsilon_2$ is always positive.

Strain invariants are properties of the strain tensor that do not change with changes in coordinate axes.

First Invariant:

$$\varepsilon_{xx}+\varepsilon_{yy}=tr(\varepsilon)=\varepsilon_1+\varepsilon_2$$

The first invariant is always the trace of the tensor (in 2D or 3D). It corresponds to the dilatational (volumetric) component of the shape change.

Second Invariant:

formally

$$\varepsilon_{xx}\varepsilon_{yy}-\varepsilon_{xy}^2$$

Another form is commonly used (C. Kreemer personal communication) which has the advantage of being in units of strain (not strain squared):

$\sqrt{\varepsilon_{xx}^2+\varepsilon_{yy}^2+2\varepsilon_{xy}^2}$ (sometimes shown squared)

This contains shear and dilatation components of deformation, and hence "invariant shear" can be accessed by taking the difference to the dilatation.

Also the difference

$$\varepsilon_1-\varepsilon_2$$

is invariant to coordinate axis changes and represents shear in the sense that if the principal strains are equal (isotropic dilatation) then the shear is zero. If $\varepsilon_1=-\varepsilon_2$ then dilatation is zero and shear is $2\varepsilon_1$.

In three dimensions the equations get a little tricker. It becomes unclear (at least to me) how the notion of a "shear invariant" works at dimensions > 2.

Models and Data

What is a model?

discussion.

What is a *good* model?

1) fits data
2) is "reasonable"
3) meets *a priori* conditions.

1) What are the measures of data fit?

Small residuals to the predictions of a model.

We can impose this condition mathematically on a problem by requiring that

$$\|\boldsymbol{r}\|=0$$

Misfit can be quantified

$$\chi^2=\sum_{i=1}^{N}\frac{(d_i-d_{i,pred})^2}{\sigma_i^2}$$

$$\chi_\nu^2=\frac{1}{\nu}\chi^2$$

also used to describe the amount of 'scatter' in the residuals:

RMS = $\sqrt{\frac{\sum r_i^2}{N}}$

and WRMS = $\sqrt{\frac{\sum w_i r_i^2}{\sum w_i}}$

which is similar (though not exactly the same) as χ^2 if using the data uncertainties as your weights.

where the "degrees of freedom" ν = number of data (N) minus number of model parameters

χ_ν^2 is always positive and follows χ^2 distributions.

2) What is reasonable?

Well that depends. But commonly we prefer simple models to complex models. This is a choice of the modeler, owing to finite human intelligence, or the belief in "Occam's razor" applied to modeling.

$$\|\boldsymbol{m}\|=0$$

3) How do we meet *a priori* conditions?

Many ways have been tried. One way is to look for a model that is close to some other particular model:

$$\|\boldsymbol{m}-\boldsymbol{m}_{apriori}\|=0$$

another choice would be to require that the model is "smooth" i.e. the norm of the Laplacian operator of **m** is as close to zero as possible.

$$\|\nabla^2\boldsymbol{m}\|=0$$

or flat

$$\|\frac{\partial\boldsymbol{m}}{\partial\boldsymbol{x}}\|=0 \quad .$$

These are just some ways and many others are possible.

In the end a model can find a balance between the three criteria by finding a way to minimize some function of all three e.g. solve the system such that

$$\|\boldsymbol{r}\| + \|\boldsymbol{m}\| + \nabla^2 \boldsymbol{m} = 0$$

the importance of the various terms can be controlled by using weights, i.e.

$$\alpha\|\boldsymbol{r}\| + \beta\|\boldsymbol{m}\| + \gamma \nabla^2 \boldsymbol{m} = 0$$

where alpha, beta, gamma are imposed by the modeler, experiented with until a model with the desired properties is found.