GEOL 695: Geophysical Geodesy - Day 5

Preliminaries

- Reading questions
- New problem set, due next week.

New Readings

- none

Uncertainty and Covariance

σ_x is the uncertainty in the estimate of x. 1-sigma vs. 2-sigma.

σ_x^2 is the estimate of variance in the estimate of x

σ_{xy} is the covariance between x and y

$$cov(\boldsymbol{V}_{xyz}) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 \end{bmatrix}$$ is the covariance matrix.

$$corr(x, y) = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$ is the correlation between x and y (see GMT psvelo)

So when rotating from xyz to ned

$$\boldsymbol{V}_{ned} = \boldsymbol{T}\,\boldsymbol{V}_{xyz}$$

where $T = \begin{bmatrix} Tnx & Tny & Tnz \\ Tex & Tey & Tez \\ Tdx & Tdy & Tdz \end{bmatrix}$

$$cov(\boldsymbol{V}_{ned}) = \boldsymbol{T}\,cov(\boldsymbol{V}_{xyz})\,\boldsymbol{T}^{\boldsymbol{T}}$$

Even if $cov(\boldsymbol{V}_{xyz})$ is diagonal, $cov(\boldsymbol{V}_{ned})$ may not be (likely won't be), so we need to take covariance into account when wanting to know uncertainties in the rotated system.

Velocity uncertainty ellipses, 95% confidence region, etc.

Most of the time the uncertainty ellipses are close to circular when looking at just the horizontal. However XYZ almost always includes some of the vertical component as well, so its important to get the vertical uncertainty (which is much larger) out by keeping track of covariance.

Strain

Two-dimensions, and three-dimensions.

The basics in changes of shape.

$$\varepsilon_{xx}=\frac{\partial u_x}{\partial x} \quad , \quad \varepsilon_{yy}=\frac{\partial u_y}{\partial y} \quad , \quad \varepsilon_{xy}=\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}$$ (sometimes over 2). Note $\varepsilon_{yx}=\varepsilon_{xy}$.

The strain (rate) tensor $\boldsymbol{\varepsilon}=\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yx} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}$ keeps track of all three.

Difference between strain and strain rate.

Velocity gradients, rotations, strain rates.

Note that on a rotating plate, velocity gradients are not zero, but strain rate is zero.

Rotation matrix in two dimensions:

Given a velocity field that is reasonably smooth, in a local patch of it we can estimate a constant strain rate tensor value. This can be separated into rotation and strain parts if the network is “strong”.

Example of line measurements.

Plane strain versus non-plane strain