GEOL 695: Geophysical Geodesy - Day 3

Announcements

Room change next time

Problem Set #1 is now ready. See class page.

Reading assignment

Readings

required:
Turcotte, D. L., and G. Schubert (1982), *Geodynamics Applications of Continuum Physics to Geological Problems*, John Wiley & Sons, Inc., New York. Sections 5-1, 5-2 , 5-3, 5-4.

optional:
Basic introduction into what is geodesy:
http://oceanservice.noaa.gov/education/kits/geodesy

Coblentz, D. D., R. M. Richardson, and M. Sandiford (1994), On the gravitational potential of the Earth's lithosphere, *Tectonics*, *13*, 929-945.

Jones, C. H., J. R. Unruh, and L. J. Sonder (1996), The role of gravitational potential energy in active deformation in the southwestern United States, *Nature*, *381*, 37-41.

Transformations

XYZ to Latitude, Longitude and Height

$$\phi = \arctan\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\left(1-e^2\frac{\nu}{\nu+h}\right)^{-1}$$ latitude

$$\lambda = \arctan\left(\frac{Y}{X}\right)$$ longitude

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\phi} - \nu$$ height

where $\nu = \frac{a}{\sqrt{(1-e^{2}\sin^{2}\phi)}}$ is “radius of curvature in the prime vertical”

and *e* is eccentricity $e^2 = f(2-f)$ and are known beforehand.

Problem: must know “h” to start so solve *iteratively*.

See also:
WGS84 Implementation Manual, Version 2.4, Eurocontrol, European Organization for the Safety of Air Navigation, Brussels, Belguim and IfEN, Institute of Geodesy and Navigation, University FAF, Munich, Germany, page 82.

XYZ rates to NED (or ENU) rates

The following example uses North, East, Down coordinates, but could be easily modified to East, North, Up. Note coordinate axes always obey right hand rule.

$$\boldsymbol{V}_{ned} = \boldsymbol{T}\,\boldsymbol{V}_{xyz}$$

where $T = \begin{bmatrix} Tnx & Tny & Tnz \\ Tex & Tey & Tez \\ Tdx & Tdy & Tdz \end{bmatrix}$

and

$$\boldsymbol{V}_{xyz} = \boldsymbol{T}^{-1}\boldsymbol{V}_{ned}$$

and

$$Tnx = \boldsymbol{n}\cdot\boldsymbol{x} = -\sin\phi\cos\lambda$$

$$Tex = \boldsymbol{e}\cdot\boldsymbol{x} = -\sin\lambda$$

$$Tdx = \boldsymbol{d}\cdot\boldsymbol{x} = -\cos\phi\cos\lambda$$

$$Tny = \boldsymbol{n}\cdot\boldsymbol{y} = -\sin\phi\sin\lambda$$

$$Tey = \boldsymbol{e}\cdot\boldsymbol{y} = \cos\lambda$$

$$Tdy = \boldsymbol{d}\cdot\boldsymbol{y} = -\cos\phi\sin\lambda$$

$$Tnz = \boldsymbol{n}\cdot\boldsymbol{z} = \cos\phi$$

$$Tez = \boldsymbol{e}\cdot\boldsymbol{z} = 0$$

$$Tdz = \boldsymbol{d}\cdot\boldsymbol{z} = -\sin\phi$$

see also Cox, A., and R. B. Hart (1986), *Plate Tectonics: How it Works*, 392 pp., Blackwell Scientific Publishing, Brookeline Village, MA. Box 4-2, p. 155-156.

XYZ displacements to NEU displacements

same just no /dt

Rotations on a Sphere

Theory of plate tectonics

From a position on surface, and Euler rotation vector

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{x}$$

picture it.

plate tectonics.

reference frames. North America, ITRF.

Cross product versus matrix forms of rotation

$$\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix} = \begin{pmatrix} -\omega_z y + \omega_y z \\ \omega_z x - \omega_x z \\ -\omega_y x + \omega_x y \end{pmatrix} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{W}\boldsymbol{x}$$

Helmert Transformation, seven parameters

- Rotation

The square 3x3 matrix ***R*** rotates a 3x1 vector into another 3x1 vector. If ***R*** is a rotation matrix it does not change the length of ***x***, only its orientation. It is, in general, an orthogonal matrix ($\boldsymbol{R}^{-1} = \boldsymbol{R}^T$) with det ***R*** = 1.

$$\boldsymbol{x}' = \boldsymbol{R}\,\boldsymbol{x}$$

one example of a rotation matrix is:

$$\boldsymbol{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix} \ ,$$

which rotates a vector around the x-axis. In general you need three numbers to specify a rotation. There are equivalents for $\boldsymbol{R}_y$ and $\boldsymbol{R}_z$ which can be multiplied together to get any particular rotation.

- Translation

This one is pretty simple, just add a vector d***x*** to ***x***

$$\boldsymbol{x}' = \boldsymbol{x} + d\,\boldsymbol{x}$$

- Scale

$$\boldsymbol{x}' = (1+s)\,\boldsymbol{x}$$

where s << 1

put it all together to get:

$$\boldsymbol{x}'=d\,\boldsymbol{x}+(1+s)\,\boldsymbol{R}\,\boldsymbol{x}$$

Note that here R has the form

$$\boldsymbol{R}=\begin{pmatrix} 1 & -r_z & r_y \\ r_z & 1 & -r_x \\ -r_y & r_x & 1 \end{pmatrix} ,$$

which is similar to the cross product matrix, but differs along the trace.

14 Parameters Helmert Transformation

Tinker Time

get hooked up with Matlab