Day 13 - October 6, 2009

Preliminaries

- Owens Lake Earthquake update
- Reading Questions
- Homework Questions
- Projects
- Field Trip - Oct 13?

New Reading

B rgmann, R., and G. Dresen (2008), Rheology of the Lower Crust and Upper Mantle: Evidence from Rock Mechanics, Geodesy, and Field Observations, *Annual Review of Earth and Planetary Science*, *36, doi:10.1146/annurev.earth.36.031207.124326*, 531-567.

Dilution of Precision

First we must see how position is found using the pseudorange observations:

$$\rho' = \rho + c\,d\tau$$

where ρ' is the observed pseudorange and ρ is the true geometric range, c is the speed of light and $d\tau$ is the receiver clock error.

for 1 station and 4 satellites we have:

$$(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 = (\rho'_i - c d\tau)^2$$

for i=1...4

now

$$\begin{bmatrix} \frac{\partial \rho_1'}{\partial x} & \frac{\partial \rho_1'}{\partial y} & \frac{\partial \rho_1'}{\partial z} & \frac{\partial \rho_1'}{\partial d\tau} \\ \frac{\partial \rho_2'}{\partial x} & \frac{\partial \rho_2'}{\partial y} & \frac{\partial \rho_{2s}'}{\partial z} & \frac{\partial \rho_2'}{\partial d\tau} \\ \frac{\partial \rho_3'}{\partial x} & \frac{\partial \rho_3'}{\partial y} & \frac{\partial \rho_3'}{\partial z} & \frac{\partial \rho_3'}{\partial d\tau} \\ \frac{\partial \rho_4'}{\partial x} & \frac{\partial \rho_4'}{\partial y} & \frac{\partial \rho_4'}{\partial z} & \frac{\partial \rho_4'}{\partial d\tau} \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \\ d\tau \end{bmatrix} = \begin{bmatrix} \rho'_1 \\ \rho'_2 \\ \rho'_3 \\ \rho'_4 \end{bmatrix}$$

where

$$\frac{\partial \rho'}{\partial x} = \frac{\partial \rho}{\partial x} = \frac{-(x_i - x)}{\rho}$$

(analogous for y and z)

$$\frac{\partial \rho'}{\partial d\tau} = c$$

so the matrix equation becomes:

$$\begin{bmatrix} \frac{(x-x_1)}{\rho_1} & \frac{(y-y_1)}{\rho_1} & \frac{(z-z_1)}{\rho_1} & c \\ \frac{(x-x_2)}{\rho_2} & \frac{(y-y_2)}{\rho_2} & \frac{(z-z_2)}{\rho_2} & c \\ \frac{(x-x_3)}{\rho_3} & \frac{(y-y_3)}{\rho_3} & \frac{(z-z_3)}{\rho_3} & c \\ \frac{(x-x_4)}{\rho_4} & \frac{(y-y_4)}{\rho_4} & \frac{(z-z_4)}{\rho_4} & c \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \\ d\tau \end{bmatrix} = \begin{bmatrix} \rho'_1 \\ \rho'_2 \\ \rho'_3 \\ \rho'_4 \end{bmatrix}$$

(obtained from http://www.gmat.unsw.edu.au/snap/gps/gps_survey/chap1/142.htm)

We can solve this system exactly, or if more satellites are available make a best estimate using our usual least-squares tools.

Notice that to get the elements of the G (design) matrix we need to have some initial (a priori) guess for ρ .

$\boldsymbol{Q} = (\boldsymbol{G}^T \boldsymbol{G})^{-1}$ which is a familiar part of the solution to the system.

$\boldsymbol{G}^{-g} = (\boldsymbol{G}^T \boldsymbol{G})^{-1} \boldsymbol{G}^T$, $\boldsymbol{m}_{est} = \boldsymbol{G}^{-g} \boldsymbol{d}$.

$\boldsymbol{Q}$ represents the geometric part of the uncertainty in position. It comes from where the satellites are in the sky.

$$\boldsymbol{Q} = \begin{bmatrix} d_x^2 & d_{xy}^2 & d_{xz}^2 & d_{xt}^{٢} \\ d_{xy}^2 & d_y^2 & d_{yz}^2 & d_{yt}^2 \\ d_{xz}^2 & d_{yz}^2 & d_z^2 & d_{zt}^2 \\ d_{xt}^2 & d_{yt}^2 & d_{zt}^2 & d_t^2 \end{bmatrix}$$

$$PDOP = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$HDOP = \sqrt{d_N^{٢} + d_E^{٢}}$$

$$VDOP=\sqrt{d_U^2}$$

$$TDOP=\sqrt{d_t^2}$$

$$GDOP=\sqrt{PDOP^2+TDOP^2}$$

HDOP and VDOP require rotating the coordinate system into NEU. These dilution of precision factors are ofter reported on the graphic user interface to handheld GPS receivers. A large DOP is bad (since its a dilution).

Various factors control how well (to what precision) a position can be estimated. These factors include how many satellites are visible, and where they are in the sky.